

Code No: R05010102

Set No. 1

I B.Tech Supplementary Examinations, Aug/Sep 2007

MATHEMATICS-I

(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the convergence of the following series $\sum \left(\frac{n^2}{2^n} + \frac{1}{n^2} \right)$ [5]
- (b) Find the interval of convergence of the series whose n^{th} term is $\sum \frac{(-1)^n (x+2)}{(2^n + 5)}$ [5]
- (c) If $a < b$ prove that $\frac{b-a}{(1+b^2)} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{(1+a^2)}$ using Lagrange's Mean value theorem. Deduce the following [6]
 - i. $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$
 - ii. $\frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$
2. (a) If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$ and $z = r \cos\theta$ prove that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2 \sin\theta$. [6]
- (b) Find the radius of curvature at any point on the curve $y = c \cosh \frac{x}{c}$. [10]
3. Trace the curve $y = a \cosh (x/a)$ and find the volume got by rotating this curve about the x-axis between the ordinates $x = \pm a$. [16]
4. (a) Form the differential equation by eliminating the arbitrary constant $\sec y + \sec x = c + x^2/2$.
- (b) Solve the differential equation:
 $(2y \sin x + \cos y) dx = (x \sin y + 2 \cos x + \tan y) dy$.
- (c) Find the orthogonal trajectories of the family: $r^n \sin n\theta = b^n$. [3+7+6]
5. (a) Solve the differential equation: $y'' - 4y' + 3y = 4e^{3x}$,
 $y(0) = -1, y'(0) = 3$.
- (b) Solve the differential equation: $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$ [8+8]
6. (a) Find the Laplace Transformation of the following function: $t e^{-t} \sin 2t$.

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- (b) Using Laplace transform, solve $y'' + 2y' + 5y = e^{-t} \sin t$, given that $y(0) = 0, y'(0) = 1$.
- (c) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$ [5+6+5]
7. (a) Prove that $\text{div}(\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \text{curl} \mathbf{A} - \mathbf{A} \cdot \text{curl} \mathbf{B}$.
- (b) Find the directional derivative of the scalar point function $\phi(x, y, z) = 4xy^2 + 2x^2yz$ at the point $A(1, 2, 3)$ in the direction of the line AB where $B = (5, 0, 4)$. [8+8]
8. Verify Stoke's theorem for the vector field $\mathbf{F} = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by the projection of the xy-plane. [16]

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Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
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1. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)\sqrt{n}}$. [5]
- (b) Find the interval of convergence of the series $\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \infty$. [5]
- (c) Write Taylor's series for $f(x) = (1-x)^{5/2}$ with Lagrange's form of remainder upto 3 terms in the interval $[0,1]$. [6]
2. (a) Locate the stationary points and examine their nature of the following functions:
 $u = x^4 + y^4 - 2x^2 + 4xy - 2y^2$, ($x > 0$, $y > 0$).
- (b) From any point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, perpendiculars are drawn to the coordinates axes. Prove that the envelope of the straight line joining the feet of these perpendiculars is the curve. $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ [8+8]
3. (a) Trace the curve $9ay^2 = x(x-3a)^2$.
- (b) Find the surface area got by rotating one loop of the curve $r^2 = a^2 \cos 2\theta$ about the initial line. [8+8]
4. (a) Form the differential equation by eliminating the constant $x^2 + y^2 - 2ay = a^2$.
- (b) Solve the differential equation $\frac{dy}{dx}(x^2 + y^3 + xy) = 1$.
- (c) If the air is maintained at 15°C and the temperature of the body cools from 70°C to 40°C in 10 minutes, find the temperature after 30 minutes. [3+7+6]
5. (a) Solve the differential equation: $(D^3 - 1)y = e^x + \sin^3 x + 2$.
- (b) Solve the differential equation: $(x^3 D^3 + 2x^2 D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$. [8+8]
6. (a) Solve the differential equation $\frac{d^2x}{dx^2} + 9x = \sin t$ using Laplace transforms given that $x(0) = 1$, $x'(0) = 0$
- (b) Change the order of integration hence evaluate $\int_0^1 \int_{x^2}^{2-x} x \, dy \, dx$ [8+8]

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7. (a) If $\phi_1 = x^2 y$ and $\phi_2 = xz - y^2$ find $\nabla \times (\nabla \phi_1 \times \nabla \phi_2)$
- (b) If $\vec{F} = (3x^2 + 6y)i - 14yzj + 20xz^2k$ evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ from $(0,0,0)$, $(1,1,1)$ along $x = t$, $y = t$, $z = t^3$. [8+8]
8. Verify divergence theorem for $F = 6zi + (2x + y)j - xk$, taken over the region bounded by the surface of the cylinder $x^2 + y^2 = 9$ included in $z = 0$, $z = 8$, $x = 0$ and $y = 0$. [16]

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Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test for convergence of the series $\sum_{\infty}^1 [\sqrt{n^4 + 1} - \sqrt{n^4 - 1}]$ [5]

(b) Find the interval of convergence of the following series
 $\frac{1}{1-x} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + \dots$ [5]

(c) Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1} \frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem. [6]
2. (a) Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid of revolution $4x^2 + 4y^2 + 9z^2 = 36$. [8+8]

(b) Find the envelope of the family of curves $\frac{ax}{\cos \alpha} - \frac{by}{\sin \alpha} = a^2 - b^2$, α is a parameter.
3. (a) Trace the curve $r = a \sin 2\theta$.

(b) Find the whole length of the curve $8a^2y^2 = x^2(a^2 - x^2)$. [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constants, $y = a \sec x + b \tan x$.

(b) Solve the differential equation $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$.

(c) If 30% of a radio active substance disappears in 10 days, how long will it take for 90% to disappear. [3+7+6]
5. (a) Solve the differential equation: $(D^2 + 4D + 4)y = 18 \cosh x$.

(b) Solve the differential equation: $(D^2 + 4)y = \cos x$. [8+8]
6. (a) Show that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} [\bar{f}(s)]$ where $n = 1, 2, 3, \dots$

(b) Evaluate: $L^{-1} \left[\frac{1}{s^2(s+2)} \right]$

(c) Evaluate $\int \int r \sin \theta \, dr \, d\theta$ over the cardioid $r = a(1 - \cos \theta)$ above the initial line. [5+6+5]

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7. (a) Evaluate $\nabla^2 \log r$ where $r = \sqrt{x^2 + y^2 + z^2}$
- (b) Find constants a, b, c so that the vector $\mathbf{A} = (x+2y+az)\mathbf{i} + (bx-3y-z)\mathbf{j} + (4x+cy+2z)\mathbf{k}$ is irrotational. Also find φ such that $\mathbf{A} = \nabla\phi$. [8+8]
8. Verify Green's theorem for $\oint_C [(3x - 8y^2)dx + (4y - 6xy)dy]$ where C is the region bounded by $x=0$, $y=0$ and $x + y = 1$. [16]

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Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) Test the convergence of the following series
 $1 + \frac{3}{1}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots \quad x > 0$ [5]
- (b) Test the following series for absolute /conditional convergence
 $\sum \frac{(-1)^n \cdot n}{3n^2 - 2}$ [5]
- (c) Expand $e^x \sec x$ as a power series in x up to the term containing x^3 [6]
2. (a) Find the points on the surface $z^2 = xy + 1$ that are nearest to the origin.
- (b) Prove that if the centre of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at one end of the minor axis lies at the other end, then the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$. [8+8]
3. (a) Trace the lemniscate of Bernoulli : $r^2 = a^2 \cos 2\theta$.
- (b) The segment of the parabola $y^2 = 4ax$ which is cut off by the latus rectum revolves about the directrix. Find the volume of rotation of the annular region. [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant : $y^2 = 4ax$.
- (b) Solve the differential equation: $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$.
- (c) In a chemical reaction a given substance is being converted into another at a rate proportional to the amount of substance unconverted. If $(1/5)^{th}$ of the original amount has been transformed in 4 minutes how much time will be required to transform one half. [3+7+6]
5. (a) Solve the differential equation $y'' - y' - 2y = 3e^{2x}$, $y(0) = 0$, $y'(0) = 2$
- (b) Solve the differential equation: $(D^2 + 1)y = \operatorname{cosec} x$ by variation of parameters method. [8+8]
6. (a) Find the Laplace Transformations of the following functions
 $e^{-3t}(2\cos 5t - 3\sin 5t)$

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(b) Find $L^{-1} \left[\log \left(\frac{s+1}{s-1} \right) \right]$

(c) Evaluate: $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx dy}{(1+x^2+y^2)}$ [5+6+5]

7. (a) Find a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4ax^2y + z^3 = 4$ cut orthogonally at $(1, -1, 2)$.
- (b) Show that $\vec{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field. Find the scalar potential and the work done by F in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$. [8+8]
8. Verify divergence theorem for $\vec{F} = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$. [16]
