

Code No: R05010102

**Set No. 1****I B.Tech Regular Examinations, Apr/May 2007****MATHEMATICS-I**

( Common to Civil Engineering, Electrical & Electronic Engineering,  
 Mechanical Engineering, Electronics & Communication Engineering,  
 Computer Science & Engineering, Chemical Engineering, Electronics &  
 Instrumentation Engineering, Bio-Medical Engineering, Information  
 Technology, Electronics & Control Engineering, Mechatronics, Computer  
 Science & Systems Engineering, Electronics & Telematics, Metallurgy &  
 Material Technology, Electronics & Computer Engineering, Production  
 Engineering, Aeronautical Engineering, Instrumentation & Control  
 Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

**Answer any FIVE Questions**  
**All Questions carry equal marks**

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1. (a) Test the convergence of the series  $\frac{2}{1} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5-9.13} + \dots \infty$ . [5]  
 (b) Find whether the following series converges absolutely / conditionally  
 $\frac{1}{6} - \frac{1}{6 \cdot 3} + \frac{1.3.5}{6.8.10} - \frac{1.3.5.7}{6.8.10.12}$ . [5]  
 (c) Prove that  $\frac{\pi}{6} + \frac{\sqrt{3}}{5} < \sin^{-1} \frac{3}{5} < \frac{\pi}{6} + \frac{1}{8}$ . [6]
2. (a) Show that the functions  $u = x+y+z$ ,  $v = x^2+y^2+z^2-2xy-2zx-2yz$  and  
 $w = x^3+y^3+z^3-3xyz$  are functionally related. Find the relation between them.  
 (b) Find the centre of curvature at the point  $(\frac{a}{4}, \frac{a}{4})$  of the curve  $\sqrt{x} + \sqrt{y} = \sqrt{a}$ .  
 Find also the equation of the circle of curvature at that point. [8+8]
3. (a) Find the length of the curve  $x^2(a^2 - x^2) = 8 a^2 y^2$ .  
 (b) Find the volume of the solid generated by revolving the lemniscates  $r^2 = a^2 \cos 2\theta$  about the line  $\theta = \frac{\pi}{2}$ . [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant :  $\log y/x = cx$ . [3]  
 (b) Solve the differential equation:  $(1 + y^2) dx = (\tan^{-1}y - x) dy$ . [7]  
 (c) The temperature of the body drops from  $100^\circ\text{C}$  to  $75^\circ\text{C}$  in ten minutes when the surrounding air is at  $20^\circ\text{C}$  temperature. What will be its temperature after half an hour. When will the temperature be  $25^\circ\text{C}$ . [6]
5. (a) Solve the differential equation:  $(D^2-1)y = x \sin x + x^2 e^x$ .  
 (b) Solve the differential equation:  $(x^2 D^2 + xD + 4)y = \log x \cos(2 \log x)$ . [8+8]
6. (a) Prove that  $L \left[ \int_t^\infty f(t) dt \right] = \int_s^\infty \bar{f}(s) ds$  where  $L [f(t)] = \bar{f}(s)$  [5]  
 (b) Find the inverse Laplace Transformation of  $\frac{3(s^2-2)^2}{2s^5}$  [6]

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- (c) Evaluate  $\iint (x^2 + y^2) dx dy$  over the area bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  [5]
7. (a) For any vector  $\mathbf{A}$ , find  $\text{div curl } \mathbf{A}$ . [6]
- (b) Evaluate  $\iint_S \mathbf{A} \cdot \mathbf{n} ds$  where  $\mathbf{A} = z \mathbf{i} + x \mathbf{j} - 3y^2 z \mathbf{k}$  and  $S$  is the surface of the cylinder  $x^2 + y^2 = 16$  included in the first octant between  $z=0$  and  $z=5$ . [10]
8. Verify Stoke's theorem for  $\mathbf{F} = -y^3 \mathbf{i} + x^3 \mathbf{j}$  in the region  $x^2 + y^2 \leq 1, z=0$ . [16]

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1. (a) Test the convergence of the following series  $\sum \left( \frac{n^2}{2^n} + \frac{1}{n^2} \right)$  [5]
- (b) Find the interval of convergence of the series whose n th term is  $\sum \frac{(-1)^n(n+2)}{(2^n+5)}$  [5]
- (c) If  $a < b$  prove that  $\frac{b-a}{(1+b^2)} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{(1+a^2)}$  using Lagrange's Mean value theorem. Deduce the following [6]
  - i.  $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$
  - ii.  $\frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$
2. (a) If  $u=x^2-y^2$ ,  $v=2xy$  where  $x=r \cos\theta$ ,  $y=r\sin\theta$ . Show that  $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$ .
- (b) For the cardioid  $r=a(1+\cos\theta)$  Prove that  $\frac{\rho^2}{r}$  is constant where  $\rho$  is the radius of curvature. [8+8]
3. (a) Find the volume of the solid generated by revolution of  $y^2 = \frac{x^3}{(2a-x)}$  about its asymptote.
- (b) Find the area of the loop of the curve  $r=a(1+\cos \theta)$ . [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant  $y = \frac{a+x}{x^2+1}$ . [3]
- (b) Solve the differential equation:  $(1-x^2)\frac{dy}{dx} - xy = y^3 \sin^{-1}x$ . [7]
- (c) Prove that the family of confocal conics  $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$  are self orthogonal ( $\lambda$  the parameter) [6]
5. (a) Solve the differential equation:  $(D^3 - 7D^2 + 14D - 8)y = e^x \cos 2x$ .
- (b) Solve the differential equation:  $(x^2 D^2 - x3D + 1)y = \frac{\log x \sin(\log x)+1}{x}$ . [8+8]
6. (a) Solve the differential equation  $\frac{d^2x}{dx^2} + 9x = \sin t$  using Laplace transforms given that  $x(0) = 1$ ,  $x(\pi/2) = 1$

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- (b) Change the order of integration hence evaluate  $\int_0^1 \int_{x^2}^{2-x} x dy dx$  [8+8]
7. (a) Prove that  $\nabla \times (\nabla \times \mathbf{A}) = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$ .
- (b) If  $\phi = 2xy^2z + x^2y$ , evaluate  $\int_C \phi \, d\mathbf{r}$  where C consists of the straight lines from (0, 0, 0) to (1, 0, 0) then to (1, 1, 0) and then to (1, 1, 1). [8+8]
8. Verify Green's theorem for  $\oint_C (y - \sin x) dx + \cos x dy$  where C is the triangle formed by the points (0,0),  $(\pi/2, 0)$  and  $(\pi/2, 1)$ . [16]

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Time: 3 hours

Max Marks: 80

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1. (a) Test the convergence of the following series  $\sum \frac{1}{(\log \log n)^n}$  [5]  
 (b) Find the interval of convergence of the series  
 $x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} + \dots$  [5]  
 (c) Show that  $\log(1 + e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \dots$  and hence deduce that  
 $\frac{e^x}{e^x + 1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \dots$  [6]
2. (a) Given that  $x + y + z = a$ , find the maximum value of  $x^m y^n z^p$ .  
 (b) Find the envelope of the circles through the origin and whose centre lies on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . [8+8]
3. (a) Trace the curve :  $r = a(1 + \cos \theta)$ .  
 (b) Find the length of the arc of the curve  $x = e^\theta \sin \theta$ ;  $y = e^\theta \cos \theta$  from  $\theta = 0$  to  $\theta = \pi/2$ . [8+8]
4. (a) Find the differential equation of all parabolas having the axis as the axis and (a,0) as the focus.  
 (b) Solve the differential equation  $\frac{x^2 dy}{dx} = e^y - x$ .  
 (c) Find the orthogonal trajectory of the family of curves  $x^{2/3} + y^{2/3} = a^{2/3}$ . [4+6+6]
5. (a) Solve the differential equation:  $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = -2 \operatorname{Cosh} x$  given that  $y(0) = 0$ ,  $y'(0) = 1$ .  
 (b) Solve the differential equation:  $(2x - 1)^3 \frac{d^3 y}{dx^3} + (2x - 1) \frac{dy}{dx} - 2y = x$ . [8+8]
6. (a) Evaluate  $L\{e^t(\cos 2t + \frac{1}{2} \sinh 2t)\}$  [5]  
 (b) Find  $L^{-1}\left[\frac{1}{s^2 + 2s + 5}\right]$  [6]  
 (c) Evaluate the triple integral  $\int_0^1 \int_y^{1-x} \int_0^{1-x-y} x \, dz \, dx \, dy$  [5]

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7. (a) Evaluate  $\nabla^2 \log r$  where  $r = \sqrt{x^2 + y^2 + z^2}$
- (b) Find constants a, b, c so that the vector  $\mathbf{A} = (x+2y+az)\mathbf{i} + (bx-3y-z)\mathbf{j} + (4x+cy+2z)\mathbf{k}$  is irrotational. Also find  $\varphi$  such that  $\mathbf{A} = \nabla\phi$ . [8+8]
8. Verify Stoke's theorem for the vector field  $\mathbf{F} = (2x-y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$  over the upper half surface of  $x^2+y^2+z^2=1$ , bounded by the projection of the xy-plane. [16]

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Time: 3 hours

Max Marks: 80

Answer any FIVE Questions  
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1. (a) Test the convergence of the series  $\frac{\sqrt{2}-1}{3^2-1} + \frac{\sqrt{3}-1}{4^2-1} + \frac{\sqrt{4}-1}{5^2-1} + \dots$  [5]
 

(b) Examine whether the following series is absolutely convergent or conditionally convergent  $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \dots$  [5]

(c) Verify Rolle's theorem for  $f(x) = \log \left[ \frac{x^2+ab}{x(a+b)} \right]$  in  $[a,b]$  ( $x \neq 0$ ). [6]
2. (a) Show that the functions  $u = x+y+z$ ,  $v = x^2+y^2+z^2-2xy-2zx-2yz$  and  $w = x^3+y^3+z^3-3xyz$  are functionally related. Find the relation between them.
 

(b) Find the centre of curvature at the point  $\left(\frac{a}{4}, \frac{a}{4}\right)$  of the curve  $\sqrt{x} + \sqrt{y} = \sqrt{a}$ . Find also the equation of the circle of curvature at that point. [8+8]
3. (a) In the evolute of the parabola  $y^2 = 4ax$ , show that the length of the curve from its cusp  $x = 2a$  to the point where it meets the parabola  $y^2 = 4ax$  is  $2a(3\sqrt{3} - 1)$ 

(b) Find the length of the arc of the curve  $y = \log \left[ \frac{e^x-1}{e^x+1} \right]$  from  $x = 1$  to  $x = 2$  [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant :  $\log y/x = cx$ . [3]
 

(b) Solve the differential equation:  $(1 + y^2) dx = (\tan^{-1}y - x) dy$ . [7]

(c) The temperature of the body drops from  $100^\circ\text{C}$  to  $75^\circ\text{C}$  in ten minutes when the surrounding air is at  $20^\circ\text{C}$  temperature. What will be its temperature after half an hour. When will the temperature be  $25^\circ\text{C}$ . [6]
5. (a) Solve the differential equation:  $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$ .
 

(b) Solve the differential equation:  $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ . [8+8]
6. (a) Find  $L [ t^2 \sin 2t ]$  [5]
 

(b) Find  $L^{-1} \left[ \frac{s+3}{(s^2-10s+29)} \right]$  [6]

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(c) Evaluate  $\int_0^{\pi/4} \int_0^{a \sin \theta} \frac{r \, dr \, d\theta}{\sqrt{a^2 - r^2}}$  [5]

7. (a) Prove that  $\nabla \times \left( \frac{\bar{A} \times \bar{r}}{r^n} \right) = \frac{(2-n)\bar{A}}{r^n} + \frac{n(\bar{r} \cdot \bar{A})\bar{r}}{r^{n+2}}$

(b) If  $\bar{F} = (x^2 - 27)i - 6yzj + 8xz^2k$  evaluate  $\int_C \bar{F} \cdot d\bar{r}$  from the point (0,0,0) to the point (1,1,1) along the straight line from (0,0,0) to (1,0,1), (1,0,0) to (1,1,0) and (1,1,0) to (1,1,1) [8+8]

8. Verify Stokes theorem  $f = x^2i - yzj + kz$  integrated around the square  $x=0, y=0, z=0, x=1, y=1$  and  $z=1$ . [16]

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