Code No: R05010102

Set No. 1

I B.Tech Supplimentary Examinations, February 2008
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test for convergence of the series $\sum_{\infty}^{1}\left[\sqrt{n^4+1}\ -\ \sqrt{n^4-1}\right]$ [5]

   (b) Find the interval of convergence of the following series
   $\frac{1}{1-x}+\frac{1}{2(1-x)^2}+\frac{1}{3(1-x)^3}+$ .... [5]

   (c) Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}\ >\ \cos^{-1}\frac{3}{5}\ >\ \frac{\pi}{3}-\frac{1}{8}$ using Lagrange's mean value theorem. [6]

2. (a) A rectangular box open at the top is to have a volume of 32 cubic ft. Find the dimensions of the box, requiring least material for its construction.

   (b) Find the radius of curvature of $y^2\ =\ x^2\ \left(\frac{a+x}{a-x}\right)$at the origin. [8+8]

3. (a) Trace the curve $9ay^2$=$(x-2a)(x-5a)^2$.

   (b) Find the entire perimeter of the cardiod r=a(1+cosθ). [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant : y^2=4ax.

   (b) Solve the differential equation: $\frac{dy}{dx}-\ \frac{\tan y}{1+x}\ =$ (1 + x) e^x sec y.

   (c) In a chemical reaction a given substance is being converted into another at a rate proportional to the amount of substance unconverted. If $(1/5)^{th}$ of the original amount has been transformed in 4 minutes how much time will be required to transform one half. [3+7+6]

5. (a) Solve the differential equation: $(D^3-7D^2+14D-8)y=e^x$ cos 2x.

   (b) Solve the differential equation: $(x^2D^2-3xD+1)y=\log x\left(\frac{\sin(\log x)+1}{x}\right)$ [8+8]

6. (a) Using Laplace transforms solve the differential equation $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$, given that x(0)=2, $x'(0)=-1$ at t = 0

   (b) Evaluate by transforming in to polar co ordinates $\iint\frac{(x^2-y^2)}{(x^2+y^2)^{3/2}}$ dydx over the region of the circle $x^2+y^2=2ax$ in the first quadrant. [8+8]

7. (a) Find the angle between the normals to the surface xy=z^3 at (4,1,2) and (3,3,-3)

   (b) Prove that the scalar field $\bar{F} = (x^2 + xy^2)\, i + (y^2 + x^2y)\, j$ is conservative and find the scalar potential [8+8]

8. (a) Apply Green's theorem to evaluate $\oint\limits_C (2xy - x^2)dx + (x^2 + y^2)dy$,
   where "C" is bounded by y = x^2 and y^2 = x.

   (b) Apply Stoke's theorem to evaluate $\int\limits_C (y\, dx\ +\ z\, dy\ +\ x\, dz)$
   where 'C' is the curve of the intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and x + z = a. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test for convergence of the series $\sum_{\infty}^{1}\left[\sqrt{n^4+1}\ -\ \sqrt{n^4-1}\right]$ [5]

   (b) Find the interval of convergence of the following series
   $\frac{1}{1-x}+\frac{1}{2(1-x)^2}+\frac{1}{3(1-x)^3}+$ .... [5]

   (c) Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}\ >\ \cos^{-1}\frac{3}{5}\ >\ \frac{\pi}{3}-\frac{1}{8}$ using Lagrange's mean value theorem. [6]

2. (a) If x+y+z=u, y+z=uv, z=uvw, then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

   (b) If ρ_1 and ρ_2 are radii of curvatures of any chord of the cardioids r=a(1+cosθ) which passes through the pole, then show that $\rho_1^2+\rho_2^2=\frac{16a^2}{9}$. [8+8]

3. (a) Trace the curve r = (2cos θ + 1).

   (b) Determine the volume of the solid generated by revolving the limacon r = a + b cosθ (a>b) about the initial line. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant : y^2=4ax.

   (b) Solve the differential equation: $\frac{dy}{dx}-\ \frac{\tan y}{1+x}$ = (1 + x) e^x sec y.

   (c) In a chemical reaction a given substance is being converted into another at a rate proportional to the amount of substance unconverted. If $(1/5)^{th}$ of the original amount has been transformed in 4 minutes how much time will be required to transform one half. [3+7+6]

5. (a) Solve the differential equation: $(D^2-6D+13)$y = 8e^{3x} sin 2x.

   (b) Solve the differential equation: $(x^2D^2+2xD)y=(x+1)^2$. [8+8]

6. (a) Find $L\left[\frac{e^{-3t}\sin 2t}{t}\right]$

   (b) Find the inverse Laplace Tranformations of $\left[\frac{4}{(s+1)(s+2)}\right]$

(c) Evaluate the triple integral $\int_{0}^{\pi/2}\int_{0}^{a\,sin\theta}\int_{1}^{\frac{a^2-r^2}{a}} r\,dzdrd\theta$ [5+5+6]

7. (a) Find the directional derivative of xy z^2+xz at (1,1,1) in the direction of i–j+2k and F is a vector

   (b) If ϕ is a scaler function, then prove that curl $\left(\phi\bar{F}\right)=\nabla\phi\times\bar{F}+\phi curl\bar{F}$ [8+8]

8. Verify Stokes theorem F=x^2i-yzj+k integrated around the square x=0, y=0, x=1,and y=1 [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)\sqrt{n}}$. [5]

   (b) Find the interval of convergence of the series $\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty$. [5]

   (c) Write Taylor's series for f(x) = $(1-x)^{5/2}$ with Lagrange's form of remainder upto 3 terms in the interval [0,1]. [6]

2. (a) If u = xyz, v = xy+yz+zx and w = x+y+z; compute $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

   (b) Find the envelope of the family of curves $y = mx + \sqrt{a^2m^2 + b^2}$ (m is a parameter). [8+8]

3. (a) Prove that the surface area generated by the revolution of the tractrix $x = a\cos t + \frac{1}{2}$ a log tan $\frac{t}{2}$, y=a sint about its asymptote is equal to the surface area of a sphere of radius 'a'.

   (b) For the cycloid x=a (θ-sinθ), y=a(1-cosθ). Find the volume of the solid generated about the tangent at the vertex. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant : y^2=4ax.

   (b) Solve the differential equation: $\frac{dy}{dx} - \frac{\tan y}{1+x}$ = (1 + x) e^x sec y.

   (c) In a chemical reaction a given substance is being converted into another at a rate proportional to the amount of substance unconverted. If $(1/5)^{th}$ of the original amount has been transformed in 4 minutes how much time will be required to transform one half. [3+7+6]

5. (a) Solve the differential equation: $(D^2 + 4D + 4)y = 18$ coshx.

   (b) Solve the differential equation: $(D^2 + 4)y =$ cosx. [8+8]

6. (a) Show that
   L{f^n(t)}= s^n $\overline{f}$(s) − s $^{n-1}$ f(0)–s^{n-2} $f'(0)$ − . . .− f $^{n-1}$(0)
   where L{f(t)}=$\overline{f}$ (s).

(b) State and prove convolution theorem to find the inverse of Laplace transforms. [8+8]

7. (a) Show that curl $(r^n \bar{r}) = 0$

   (b) Evaluate $\iint\limits_S \bar{F}.\bar{n}\, ds$ where $\bar{F}$=zi+xj+3y^2zk where S is the surface of the cylinder x^2+y^2=1 in the first octant between z=0 and z=2 [8+8]

8. (a) Use Stokes theorem to evaluate $\int\limits_C (ydx + zdy + xdz)$, where 'C' is the curve of intersection of $x^2+y^2+z^2=a^2$ and x+z=a.

   (b) Using Green's theorem evaluate $\oint\limits_C (2x^2 - y)\, dx + (x^2 + y^2)\, dy$ where C is the boundary in xy -plane of the arc enclosed by the x-axis the semi circle $x^2 + y^2 = 1$ in the upper half of the xy- plane. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series
$\frac{x}{1} + \frac{1.x^3}{2.3} + \frac{1.3x^5}{2.4.5} + \frac{1.3.5x^7}{2.4.6.7} + \ldots.$ $(x > 0)$. [5]
   (b) Test whether the following series is absolutely convergent/ conditionally convergent. $\sum_{1}^{\infty} (-1)^{n+1} \left(\sqrt{n+1} - \sqrt{n}\right)$. [5]
   (c) Verify Lagrange's mean value theorem f(x) = $log_e x$ in [1,e]. [6]

2. (a) Locate the stationary points and examine their nature of the following functions:
u = $x^4 + y^4 - 2x^2 + 4xy - 2y^2$, (x > 0, y > 0).
   (b) From any point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, perpendiculars are drawn to the coordinates axes. Prove that the envelope of the straight line joining the feet of these perpendiculars is the curve. $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1$ [8+8]

3. (a) Find the length of the curve $x^2(a^2 - x^2) = 8\,a^2y^2$.
   (b) Find the volume of the solid generated by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the line $\theta = \frac{\pi}{2}$. [8+8]

4. (a) Obtain the differential equation of the family: y = c sinx + 2 $\sin^2$x.
   (b) Solve the differential equation: $\frac{dy}{dx}$ + y tan x = y^2sec x.
   (c) If the air is maintained at 30 ^{0}C and the temperature of a body cools from 80 ^0C to 60 ^0C in 12 minutes, find the temperature of the body after 24 minutes. [3+7+6]

5. (a) Solve the differential equation: $(D^4 - 5D^2 + 4)y = 10\ cosx$.
   (b) Solve the differential equation: $(D^2 + 5D + 4)y = x^2$. [8+8]

6. (a) Solve the differential equation $(D^2 - 3D + 2)y = e^{2x}$ given that
y(0) = − 3y′ (0) = 5 using Laplace transforms.

(b) Evaluate $\int_0^3 \int_1^2 xy(x+y)\; dx\, dy$ [10+6]

7. (a) Prove that $\nabla(\mathbf{A}.\mathbf{B})=(\mathbf{B}.\nabla)\mathbf{A}+(\mathbf{A}.\nabla)\mathbf{B}+\mathbf{B}\times(\nabla\times\mathbf{A})+\mathbf{A}\times(\nabla\times\mathbf{B})$.

   (b) If $\phi = 2xy^2z + x^2y$, evaluate $\int_C \phi\, d\mathbf{r}$ where C is the curve $x = t, y = t^2, z = t^3$ from t=0 to t=1. [8+8]

8. Verify Stoke's theorem for $\mathbf{F} = -y^3\mathbf{i}+x^3\mathbf{j}$ in the region $x^2+y^2 \leq 1$, z=0. [16]

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