Code No: R05010102

Set No. 1

I B.Tech Semester Supplimentary Examinations, June 2009
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\frac{2}{1} + \frac{2.5.8}{1.5.9} + \frac{2.5.8.11}{1.5.9.13} + ...\infty$. [5]
   (b) Find whether the following series converges absolutely / condtionally $\frac{1}{6} - \frac{1}{6}.\frac{3}{8} + \frac{1.3.5}{6.8.10} - \frac{1.3.5.7}{6.8.10.12}$. [5]
   (c) Prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{5} < \sin^{-1}\frac{3}{5} < {}^{\pi}\!/_{6} + \frac{1}{8}$. [6]

2. (a) Find the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$
   (b) if $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ [8+8]

3. Show that the total length of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ is $4\frac{\left(a^2+ab+b^2\right)}{ab}$. Hence or otherwise find the whole length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Show also that in the last curve $s^3 \alpha x^2$. [16]

4. (a) Find the differential equation of the family of cardiods r = a (1 + cos θ).
   (b) Solve the differential equation: $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}$
   (c) A copper ball is heated to 100 ^{0}C temperature. Then at time t = 0 it is placed in water that is maintained at a temperature of 30 ^{0} C. At the end of 3 minutes the temperature of the ball is reduced to 70 ^{0}C. Find the time at which the temperature of the ball is reduced to 31^0 C. [3+7+6]

5. (a) Solve the differential equation: y$''$ + 4y$'$ + 20y = 23 sint - 15cost, y(0) = 0, y$'$(0) = -1.
   (b) Using variation of parameters method solve y$''$ + 4y = 4 sec^2 2x. [8+8]

6. (a) Find the Laplace Transformation of the rectified sine-wave function defined by
   $$f(t) = \begin{cases} sin\ ,\omega t & 0 < t < \frac{\pi}{\omega} \\ O\ , & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$
   (b) Find $L^{-1}\left[\frac{S^2+2S-4}{(S^2+9)(S-5)}\right]$

(c) Change the order of integration and evaluate $\int_{0}^{1}\int_{x^2}^{2-x} xy\,dxdy$. [5+6+5]

7. (a) Find $\overline{A}.\nabla\phi$ at (1, −1, 1) if $\overline{A}$ $=3xyz^2i + 2xy^3j - x^2yzk$ and $\phi = 3x^2 - yz$.

   (b) Show that **F**$=(2xy + z^3)i + x^2j + 3xz^2k$ is a conservative force field. Find the scalar potential. Find the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4). [8+8]

8. Verify Stoke's theorem for **F** = (y-z+2) **i**+(yz+4) **j** - xz**k** where S is the surface of the cube x=0, y=0, z=0, x=2, y=2, z=2 above the xy-plane. [16]

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Set No. 2

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the series
$\frac{x}{1} + \frac{1.x^3}{2.3} + \frac{1.3x^5}{2.4.5} + \frac{1.3.5x^7}{2.4.6.7} + \ldots.\ (x > 0)$. [5]
   (b) Test whether the following series is absolutely convergent/ conditionally convergent. $\sum_{1}^{\infty} (-1)^{n+1} \left(\sqrt{n+1} - \sqrt{n}\right)$. [5]
   (c) Verify Lagrange's mean value theorem f(x) = $log_e x$ in [1,e]. [6]

2. (a) Suppose a closed rectangular base has length twice its breadth and has constant volume (V). Determine the dimensions of the base requiring least surface area (sheet metal).
   (b) Show that the equation of the evolute to the parabola
x^2=4ay is $4(y-2a)^3 = 27ax^2$. [8+8]

3. (a) Trace the curve $xy^2 = a^2(a - -x)$
   (b) Find the volume of the solid generated by the revolution of one arch of the cycloid x-a(θ + sin θ) y=a(1+cosθ) about its base. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant :
log y/x = cx.
   (b) Solve the differential equation: (1+ y^2) dx = (tan $^{-1}$y – x) dy.
   (c) The temperature of the body drops from 100 ^{0}C to 75 ^{0}C in ten minutes when the surrounding air is at 20 ^{0}C temperature. What will be its temperature after half an hour. When will the temperature be 25 ^{0}C. [3+7+6]

5. (a) Solve the differential equation: (D^2-1)y= $x$sin$x$ + x^2 e^x.
   (b) Solve the differential equation: (x^2D^2+xD+4)y=log x cos (2log$x$). [8+8]

6. (a) Find the Laplace Transformation of the rectified sine-wave function defined by
$$f(t) = \begin{cases} sin\ , \omega t & 0 < t < \frac{\pi}{\omega} \\ O\ , & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

(b) Find $L^{-1}\left[\frac{S^2+2S-4}{(S^2+9)(S-5)}\right]$

(c) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dxdy$. [5+6+5]

7. (a) Find the angle between the tangent planes to the surface x log z = y^2 – 1, x^2y = 2 – z at the point (1, 1, 1).

   (b) Evaluate $\int_S \overline{F}\,.\,N\,ds$ where F = (x + y^2) i – 2x j + 2 yz k and S is the surface of the plane 2x + y + 2z = 6 in the first octant. [8+8]

8. Verify divergence theorem for **F** = 4x**i** – 2y^2**j** + z^2**k** taken over the surface bounded by the region x^2+y^2=4, z = 0 and z = 3. [16]

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Code No: R05010102 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the series $\sum \frac{n^4}{n!}$. [5]
   (b) Find the whether the series $\frac{1}{6} - \frac{1.3}{6.8} + \frac{1.3.5}{6.8.10} - \frac{1.3.5.7}{6.8.10.12} + ...$ converges absolutely or conditionally. [6]
   (c) Verify Rolle's theorem for $f(x) = x^{2n-1}$ (a - x)^{2n} in (0,a). [5]

2. (a) Find the shortest distance from origin to the surface $xyz^2 = 2$.
   (b) Find the evolute of the hyperbola $x^2/a^2 - y^2/b^2 = 1$. Deduce the evolute of a rectangular hyperbola. [8+8]

3. (a) Prove that the surface area generated by the revolution of the tractrix $x = a\cos t + \frac{1}{2}$ a log tan $\frac{t}{2}$, y=a sint about its asymptote is equal to the surface area of a sphere of radius 'a'.
   (b) For the cycloid x=a (θ-sinθ), y=a(1-cosθ). Find the volume of the solid generated about the tangent at the vertex. [8+8]

4. (a) Form the differential equation by eliminating the parameter 'a' from the equation:
   $x^2+y^2 + 2ax + 4 = 0$.
   (b) Solve the differential equation:
   $(x^2 - 2xy + 3y^2)\,dx + (y^2 + 6xy - x^2)\,dy = 0$.
   (c) The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What is the value of N after 1 ½ hour. [3+7+6]

5. (a) Solve the differential equation: $(D^2 + 1)y = e^{-x} + x^3 + e^x$ sinx.
   (b) Solve $(D^2 + 4)y = sec2x$ by the method of variation of parameters. [8+8]

6. (a) Find the Laplace Transformation of the rectified sine-wave function defined by
   $$f(t) = \begin{cases} sin\ ,\omega t & 0 < t < \frac{\pi}{\omega} \\ O\ , & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

(b) Find $L^{-1}\left[\frac{S^2+2S-4}{(S^2+9)(S-5)}\right]$

(c) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dxdy$. [5+6+5]

7. (a) Prove that $\nabla\left[\nabla \cdot \left(\frac{\overline{r}}{r}\right)\right] = -\frac{2\,\overline{r}}{r^3}$

   (b) Find the work done in moving a particle in the force field $\overline{F} = 3x^2\, i + (2xz - y)\, j + z\, k$ along the curve $x^2 = 4y$, $3x^3 = 8z$ from x = 0 to x = 2. [8+8]

8. (a) Apply Stoke's theorem to evaluate $\int_C ((x+y)dx + (2x-3)\,dy + (y+z)dz)$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6).

   (b) Evaluate by Green's theorem $\oint_C [(\cos x\ \sin y - 2\,x\,y)\,dx + \sin x \cos y\, dy]$ where 'C' is the circle $x^2 + y^2 = 1$. [8+8]

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Code No: R05010102 **Set No. 4**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\frac{\sqrt{2}-1}{3^2-1}+\frac{\sqrt{3}-1}{4^2-1}+\frac{\sqrt{4}-1}{5^2-1}+$ ..... [5]
   (b) Examine whether the following series is absolutely convergent or conditionally convergent $1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\ldots\ldots$ [5]
   (c) Verify Rolle's theorem for f(x) = log $\left[\frac{x^2+ab}{x(a+b)}\right]$ in [a,b] (x $\neq$ 0). [6]

2. (a) Find the shortest distance from origin to the surface xyz^2 = 2.
   (b) Find the evolute of the hyperbola x^2/a^2–y^2/b^2 = 1. Deduce the evolute of a rectangular hyperbola. [8+8]

3. (a) Prove that the surface area generated by the revolution of the tractrix $x = a\cos t+\frac{1}{2}$ a log tan $\frac{t}{2}$, y=a sint about its asymptote is equal to the surface area of a sphere of radius 'a'.
   (b) For the cycloid x=a (θ-sinθ), y=a(1-cosθ). Find the volume of the solid generated about the tangent at the vertex. [8+8]

4. (a) Obtain the differential equation of the co-axial circles of the system x^2+y^2+2ax + c^2 = 0 where c is a constant and a is a variable parameter.
   (b) Solve the differential equation: $\frac{dy}{dx}=\frac{x-y\cos x}{1+\sin x}$
   (c) Find the orthogonal trajectories of the co-axial curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, λ being a parameter [3+7+6]

5. (a) Solve the differential equation: (D^2+2D–3) y=x^2e^{-3x}.
   (b) Solve the differential equation: (D^2+4)=sec2x by the method of variation of parameters. [8+8]

6. (a) Find $L\left[\frac{\sinh 2t}{t}\right]$
   (b) Find $L^{-1}\left[\frac{s}{(s-3)(s^2+4)}\right]$

(c) Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in first quadrant. [5+5+6]

7. Prove that $\mathbf{F}=(y^2\cos x+z^3)i+(2y\sin x-4)j+(3xz^2+2)k$ is a conservative force field. Find the work done in moving an object in this field from (0, 1, –1) to (π/2, –1, 2). [16]

8. (a) Apply Stoke's theorem to evaluate $\int_C((x+y)dx+(2x-3)\,dy+(y+z)dz)$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6).

(b) Evaluate by Green's theorem $\oint_C[(cos\,x\ sin\,y-2\,x\,y)\,dx+sinx\,cosy\,dy]$ where 'C' is the circle $x^2+y^2=1$. [8+8]

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