2. $\lambda_{max}/2 = 40 \Rightarrow \lambda_{max} = 80$

4. Large aperture increases the amount of light gathered by the telescope increasing the resolution.

5. $KE = \frac{1}{2}mv_{esc}^2 = \frac{1}{2}m(\sqrt{2gR})^2 = mgR$

6. A voltmeter is a high resistance galvanometer and is connected in parallel to circuit and ammeter is a low resistance galvanometer so if we connect high resistance in series with ammeter its resistance will be much high.

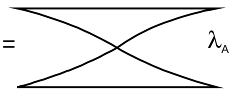
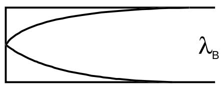
7. In coil A, $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}$. $\quad \therefore B \alpha \frac{I}{R}$; Hence, $\frac{B_1}{B_2} = \frac{I_1}{R_1}.\frac{R_2}{I_2} = \frac{2}{2} = 1$

8. No. of images, n = $(360/\theta) - 1$. As $\theta = 60^0$ so $n = 5$

9. $P_1 = V^2/R$; $P_2 = \frac{V^2}{(R/2)} + \frac{V^2}{(R/2)} = 4\frac{V^2}{R} = 4P_1$

10. $E_n = -\frac{13.6}{n^2} \Rightarrow E_2 = -\frac{13.6}{2^2} = 3.4eV$

11. $\frac{\lambda_A}{\lambda_B} = \frac{1}{2} \Rightarrow \frac{n_A}{n_B} = \frac{2}{1}$

A =  $\lambda_A = 2L$ B =  $\lambda_B = 4L$

12. The fact that placing wax decreases the frequency of the unknown fork and also the beat frequency states that the unknown fork is of higher frequency.
n - 288 = 4 $\Rightarrow$ n = 292 cps

13. $y_1 + y_2 = a\sin(\omega t - kx) - a\sin(\omega t + kx)$

$= -2a\cos\omega t \times \sin kx \Rightarrow y_1 + y_2 = 0$ at $x = 0$

14. W = qV $\Rightarrow V_A - V_B$ = 2/20 = 0.1 V
Here W is the work done in moving charge q from point A to B

15. r = mv / Bq is same for both

16. K.E. is maximum and P.E minimum at mean position

17. Angular momentum = conserved

$$\frac{1}{2}MR^2\omega_1 = 2mR^2\omega + \frac{1}{2}MR^2\omega \Rightarrow \omega = \frac{M\omega_1}{M+4m}$$

18. The condition to avoid skidding, $v = \sqrt{\mu rg} = \sqrt{0.6 \times 150 \times 10} = 30\ m/s$

19. v = $\sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20\ m/s$

20. $W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} Kx dx = K\left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \frac{K}{2}[x_2^2 - x_1^2] = \frac{800}{2}[(0.15)^2 - (0.05)^2] = 8\,J$

21. Conserving Linear Momentum

$2Mv_c = 2Mv - Mv \Rightarrow v_c = v/2$

22. It will compress due to the force of attraction between two adjacent coils carrying current in the same direction

24. Semiconductors are insulators at low temperature

27. Neutrons can't be deflected by a magnetic field

28. $hc/\lambda_0 = W_0$; $\frac{(\lambda_0)_1}{(\lambda_0)_2} = \frac{(W_0)_2}{(W_0)_1} = \frac{4.5}{2.3} = 2:1$

29. Covalent bond formation is best explained by orbital theory which uses wave phenomena

32. Amount left = $N_0/2^n = N_0/8$ (Here n = 15/5 = 3)

33. Use $R_t = R_0\left(\frac{T}{273}\right)$

34. $E = \sum \frac{1}{2}CV^2 = \frac{1}{2}nCV^2$

35. Black body also emits radiation whereas nothing escapes a black hole.

36. The given circuit clearly shows that the inductors are in parallel we have, $\frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ or $L = 1$

37. As the velocity at the highest point reduces to zero. The K.E. of the ball also becomes zero.

38. As the ball moves down from height 'h' to ground the P.E at height 'h' is converted to K.E. at the ground (Applying Law of conservation of Energy)

Hence, $\frac{1}{2}m_A v_A^2 = m_A g h_A$ or $v_A = \sqrt{2gh_A}$; Similarly, $v_B = \sqrt{2gh}$ or $v_A = v_B$

39. Let the initial velocity of the body be v. Hence the final velocity = v/2

Applying $v^2 = u^2 - 2as \Rightarrow \left(\frac{v}{2}\right)^2 = v^2 - 2.a.3 \Rightarrow a = v^2/8$

In IInd case when the body comes to rest, final velocity = 0, initial velocity = $\frac{v}{2}$

Again, $(0)^2 = \left(\frac{v}{2}\right)^2 - 2.\frac{v^2}{8}.s$; or $s = 1cm$

So the extra penetration will be 1 cm

40. When gravitational force becomes zero so centripetal force on satellite becomes zero so satellite will escape its round orbit and becomes stationary.

41. The molecular kinetic energy increases, and so temperature increases.

43. Because thermal energy decreases, therefore mass should increase

44. Maximum in insulators and overlapping in metals

46. $E = (PE)_{final} - (PE)_{initial} = \frac{-GMm}{3R} + \frac{GMm}{R} = \frac{GMm}{6R}$

47. Spring constant becomes n times for each piece. $T = 2\pi\sqrt{m/k}$

$\frac{T_1}{T_2} = \frac{\sqrt{nK}}{K}$ or $T_2 = T/\sqrt{n}$

48. The flux for both the charges exactly cancels the effect of each other

49. W = $\frac{V^2}{R_{net}}$; $150 = \frac{(15)^2}{R} + \frac{(15)^2}{2} \Rightarrow R = 6\Omega$

50. Resolving power $\alpha(1/\lambda)$. Hence, $\frac{(R.P)_1}{(R.P)_2} = \frac{\lambda_2}{\lambda_1} = \frac{5}{4}$

51. $T = 2\pi\sqrt{l_{eff}/8}$; l_{eff} decreases when the child stands up.

52. Man in the lift is in a non - inertial frame so we have to take into account the pseudo acceleration

53. From Faradays law of electrolysis, $m \propto it$.

54. $v_{rms}\alpha\sqrt{T/m}$; $\sqrt{\frac{273+47}{32}} = \sqrt{\frac{T}{2}}$ or $T = 20K$

55. $T = 2\pi m/Bq$

57. $I_1N_1 = I_2N_2 \Rightarrow I_2 = \frac{4\times140}{280} = 2A$

58. Absolute zero temperature is practically not reachable

60. Resultant of F_2 and F_3 is of magnitude F_1.

61. Use $\tan\alpha = \frac{P\sin\theta}{Q+P\cos\theta} \Rightarrow \tan90^0 = \frac{P\sin\theta}{Q+P\cos\theta} = \infty \therefore Q + P\cos\theta = 0 \Rightarrow P\cos\theta = -Q$

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$ $R = \sqrt{P^2+Q^2-2Q^2}$ or $R = \sqrt{P^2-Q^2} = 12$

144 = (P + Q) (P - Q) or P - Q = 144/18 = 8 $\therefore$ P = 13 N and Q = 5 N

62. Use $u^2 = 2as$. a is same for both cases

$s_1 = u^2/2a$; $s_2 = 16\,u^2/2a = 16\,s_1 \Rightarrow s_1 : s_2 = 1 : 16$

63. γ for resulting mixture should be in between 7/5 and 5/3

64. Apply the condition for equilibrium of each charge

65. $4\pi\in_0 R = 1.1\times10^{-10}$

66. $a = \frac{m_1-m_2}{m_1+m_2}g$; $\frac{1}{8} = \frac{m_1-m_2}{m_1+m_2} \Rightarrow m_1 : m_2 = 9 : 7$

67. Energy radiated αR^2T^4

68. Apply Newton's second law

$F - T_{ab} = ma$; $T_{ab} - T_{bc} = ma$ $\therefore$ $T_{bc} = 7.8$ N

69. T - 60 g = 60 a; T = 3000 N ; $\therefore$ a = 4 ms^{-2}

70. Zero, line of motion through the point P.

72. $v_{esc} = \sqrt{2gR}$, where R is radius of the planet

Hence escape velocity is independent of m

73. β - rays are the beam of fast moving electrons

74. Both have the dimension $M^1L^2T^{-2}$

80. The nitro group can attach to metal through nitrogen as ($-NO_2$) or through oxygen as nitrito (-ONO)

81. - CH_3 group has + I effect , as number of - CH_3 group increases, the inductive effect increases.

82. Bond between C of organic molecule and metal atom.

84. $(HSO_4)^-$ can accept and donate a proton

$(HSO_4)^- + H^+ \rightarrow H_2SO_4$

$(HSO_4)^- - H^+ \rightarrow SO_4^{2-}$

85. $Mg(OH)_2 \rightarrow [Mg^{2+}] + 2[OH^-]$

x 2x

$K_{sp} = [Mg]\ [OH]^2 = [x]\ [2x]^2 = x\ .\ 4x^2 = 4x^3$

86. K = $(mol\ L^{-1})^{1-n}\ sec^{-1}$, n = 0,1

87.

XeF_2	$sp^3\ d$	3 lone pairs
XeF_4	sp^3d^2	2 lone pairs
XeF_6	sp^3d^3	1 lone pair

89. Order is the sum of the power of the concentrations terms in rate law expression.

91. According to bond order values the given order is the answer. Bond order values are $+1, +1\frac{1}{2}, +2$ and $+2\frac{1}{2}$, higher bond order means stronger bond.

92. ΔH +ve at low temperature and ΔS +ve at low temperature shows that reaction is non spontaneous

At high temperature (boiling point) becomes feasible

93. Some mechanical energy is always converted (lost) to other forms of energy.

95. According to their positions in the periods, these values are in the order :

$Yb^{+3} < Pm^{+3} < Ce^{+3} < La^{+3}$

At nos. 70 61 58 57

This is due to lanthanide contraction

96. KO_2 is a very good oxidising agent

$_7N = 1s^2\ 2s^22p^3$; $_{15}P = 1s^22s^22p^63s^23p^3$

In phosphorous the 3d - orbitals are available,

100. PV = nRT (number of moles = n/V) $\therefore$ n/V = P/RT

103. NH_4^+ ions are increased to suppress release of OH^- ions, hence solubility product of $Fe(OH)_3$ is attained. Colour of precipitate is different.

104. According to molecular weight given

107. 2nd excited state will be the 3rd energy level

$$E_n = \frac{13.6}{n^2} eV \text{ or } E = \frac{13.6}{9} eV = 1.51 eV$$

110. $CH_3CH_2COOH \xrightarrow[\text{red P}]{Cl_2} CH_3CHClCOOH \xrightarrow[-HCl]{\text{alc. KOH}} \underset{\text{Acrylic acid}}{CH_2 = CHCOOH}$

111. Alumina is mixed with cryolite which acts as an electrolyte

112. Silver ore forms a soluble complex with NaCN from which silver is precipitated using scrap zinc.

$$Ag_2S + 2NaCN \rightarrow Na[Ag(CN)_2] \xrightarrow{Zn} Na_2[Zn(CN)_4] + Ag\downarrow$$

Sod. argento - cyanide (soluble)

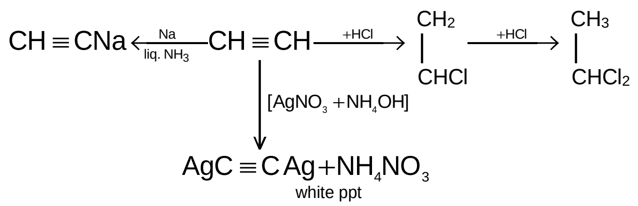
114. $\Delta T_b = K_b \times \frac{W_B}{M_B \times W_A} \times 1000$; $\Delta T_f = K_f \frac{W_B}{M_B \times W_A} \times 1000$; $\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f} = \frac{\Delta T_b}{-0.186} = \frac{0.512}{1.86} = 0.0512^0 C$

115. E_{cell} = Reduction potential of cathode (right) - reduction potential of anode (left)

$= E_{right} - E_{left}$

116. $\Delta x.\Delta v = \frac{h}{2\pi m}$

117. Acetylene reacts with the other three as

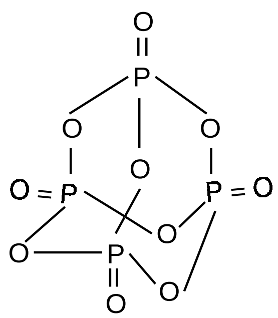


118. In this reaction the ratio of number of moles of reactants to products is same i.e. 2 : 2, hence change in volume will not alter the number of moles.

119. ΔH negative shows that the reaction is spontaneous. Higher value for Zn shows that the reaction is more feasible.

120. Mn^{2+} has the maximum number of unpaired electrons (5) and therefore has maximum moment.

121. In molecules (a), (c) and (d), the carbon atom has a multiple bond, only (b) has sp^3 hybridisation

124. 

126. Beryllium shows anomalous properties due to its small size

127. $E_{cell} = E_{right\ (cathode)} - E_{left\ (anode)}$

128. $CH \equiv CH + \xrightarrow{HOCl} \begin{matrix} CHOH \\ || \\ CHCl \end{matrix} \xrightarrow{HOCl} \left[\begin{matrix} CH(OH)_2 \\ | \\ CHCl_2 \end{matrix}\right] \xrightarrow{-H_2O} \begin{matrix} CHO \\ | \\ CHCl_2 \end{matrix}$

dichloroacetaldehyde

129. Aldehydic group gets oxidised to carboxylic group

Double bond breaks and carbon gets oxidised to carboxylic group

130 The E^0 of cell will be zero

132. $C_2H_5NH_2 + CHCl_3 + 3KOH \rightarrow C_2H_5N \equiv C + 3KCl + 3HCl$

Ethyl isocyanide

135. After every 5 years amount is becoming half.

$\therefore 64g \xrightarrow{5yrs} 32g \xrightarrow[(10)]{5yrs} 16g \xrightarrow[(15)]{5\ yrs} 8g$

after 15 years.

136. Forms a soluble complex which is precipitated with zinc

138. Volume increases with rise in temperature.

141. Pure metal always deposits at cathode

142. A more basic ligand forms stable bond with metal ion, Cl^- is most basic amongst all

143. ${}_0n^1 \rightarrow {}_{+1}p^1 + {}_{-1}e^0$

144. $[\Delta H_{mix} < 0]$

146. BCC - points are at corners and one in the centre of the unit cell

Number of atoms per unit cell = $8 \times \frac{1}{8} + 1 = 2$

FCC - points are at the corners and also centre of the six faces of each cell

Number of atoms per unit cell = $8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

147. Fe (no. of moles) = $\frac{558.5}{55.85} = 10\text{moles}$

C (no. of moles) = 60/12 = 5 moles.

148. $\overset{+3}{Mn_2}O_3 \xleftarrow{-4e^-} [K\overset{+7}{Mn}O_4] \xrightarrow{-e^-} [MnO_4]^{-1}$

$[K\overset{+7}{Mn}O_4] \xrightarrow{-5e^-} Mn^{2+}$

$[K\overset{+7}{Mn}O_4] \xrightarrow{-3e^-} \overset{+4}{Mn}O_2$

149. The oxidation states show a change only in reaction (d)

$\overset{0}{Zn} + w\overset{+1}{A}gCN \longrightarrow 2\overset{0}{A}g + \overset{+2}{Zn}(CN)_2$

150. $K_p = K_c(RT)^{\Delta n}; \Delta n = 1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2}$

$\therefore \frac{K_p}{K_c} = (RT)^{-1/2}$

AIEEE - 2002
Mathematics Solution

1. We have $\alpha^2 = 5\alpha - 3$

$\Rightarrow \alpha^2 - 5\alpha + 3 = 0 \Rightarrow \alpha = \frac{5 \pm \sqrt{13}}{2}$. Similarly, $\beta^2 = 5\beta - 3 \Rightarrow \alpha = \frac{5 \pm \sqrt{13}}{2}$

$\therefore \alpha = \frac{5+\sqrt{13}}{2}$ and $\beta = \frac{5-\sqrt{13}}{2}$ or vice - versa

$\alpha^2 + \beta^2 = \frac{50+26}{4} = 19$ & $\alpha\beta = \frac{1}{4}(25-13) = 3$

Thus, the equation having $\frac{\alpha}{\beta}$ & $\frac{\beta}{\alpha}$ as its roots is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0 \Rightarrow x^2 - x\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right) + 1 = 0 \text{ or } 3x^2 - 19x + 1 = 0$$

2. $y = (x + \sqrt{1+x^2})^n$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{1}{2}(1+x^2)^{-1/2}.2x\right); \frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}} = \frac{n(\sqrt{1+x^2}+x)^n}{\sqrt{1+x^2}}$$

or $\sqrt{1+x^2}\frac{dy}{dx} = ny$ or $\sqrt{1+x^2}y_1 = ny$ $\left(y_1 = \frac{dy}{dx}\right)$. Squaring, $(1+x^2)y_1^2 = n^2y^2$

Differentiating, $(1+x^2)\,2y_1y_2 + y_1^2 \cdot 2x = n^2 \cdot 2yy_1$ (Here, $y_2 = \frac{d^2y}{dx^2}$) or $(1+x^2)y_2 + xy_1 = x^2y$

3. 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P.
$\Rightarrow 2\log_9(3^{1-x}+2) = 1 + \log_3(4.3^x-1)$
$\log_3(3^{1-x}+2) = \log_3 3 + \log_3(4.3^x-1)$
$\log_3(3^{1-x}+2) = \log_3[3(4.3^x-1)]$
$3^{1-x} + 2 = 3(4.3^x-1)$ (put $3^x = t$)

$\frac{3}{t} + 2 = 12t - 3$ or $12t^2 - 5t - 3 = 0$

Hence $t = -\frac{1}{3}, \frac{3}{4} \Rightarrow 3^x = \frac{3}{4} \Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x = \log_3 3 - \log_3 4 \Rightarrow x = 1 - \log_3 4$

4. $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{3}$ and $P(E_3) = \frac{1}{4}$; $P(E_1 \cup E_2 \cup E_3) = 1 - P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$

$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) = 1 - \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{3}{4}$

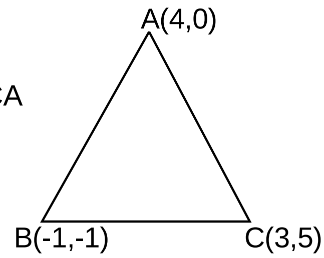
5. $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; Period $= \frac{2\pi}{2} = \pi$

6. $l = AR^{p-1} \Rightarrow \log l = \log A + (p-1)\log R$
$m = AR^{q-1} \Rightarrow \log m = \log A + (q-1)\log R$
$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1)\log R$
Now,

$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (q-1)\log R & r & 1 \end{vmatrix} = 0$$

7. $$\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x} \Rightarrow \lim_{x\to 0}\frac{\sqrt{1-(1-2\sin^2 x)}}{\sqrt{2}x}\ ;\ \lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x} \Rightarrow \lim_{x\to 0}\frac{|\sin x|}{x}$$
the function does not exist or LHS $\neq$ RHS

8. $AB = \sqrt{(4+1)^2+(0+1)^2} = \sqrt{26}$; $BC = \sqrt{(3+1)^2+(5+1)^2} = \sqrt{52}$



CA $= \sqrt{(4-3)^2+(0-5)^2} = \sqrt{26}$; So, in isosceles triangle AB = CA
For right angled triangle $BC^2 = AB^2 + AC^2$
So, here BC = $\sqrt{52}$ or $BC^2 = 52$ or $(\sqrt{26})^2 + (\sqrt{26})^2 = 52$
So, given triangle is right angled and also isosceles

9. Total student = 100 ; for 70 stds $75 \times 70 = 5250 \Rightarrow 7200 - 5250 = 1950$

Average of girls = $\dfrac{1950}{30} = 65$

10. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x \Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1+\frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = x \Rightarrow \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} \text{ OR } \cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha} \text{ or cosec } x = \frac{1+\cos\alpha}{1-\cos\alpha}$$

$$\sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1-(1-2\sin^2\alpha/2)}{1+2\cos^2\alpha/2-1} \text{ or } \sin x = \tan^2\frac{\alpha}{2}$$

11. Order = 3, degree = 3

12. $\dfrac{x-4}{1} = \dfrac{y-7}{5} = \dfrac{z-4}{4}$ ....(i)
a(x-4) + b(y - 7) + c (z - 4) = 0 .... (ii)
Line passing through point (3, 2, 0)
a + 5x + 4 c + 0 .... (iii)
Solving the equation we get by equation (ii)
x - y + z = 1

13. $$\frac{d^2y}{dx^2} = e^{-2x};\ \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c\ ;\ y = \frac{e^{-2x}}{4} + cx + d$$

14. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{\frac{1}{x}} = \lim_{x\to\infty}\left(\frac{1+\frac{5}{x}+\frac{3}{x^2}}{1+\frac{1}{x}+\frac{3}{x^2}}\right)^{\frac{1}{x}} = 1$

15. $f(x) = \sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right)$ exists if

$-1 \le \log_3\left(\frac{x}{3}\right) \le 1 \Leftrightarrow 3^{-1} \le \frac{x}{3} \le 3^1 \Leftrightarrow 1 \le x \le 9$ or $x \in [1, 9]$

17. $ar^4 = 2$
$a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$
$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

18. $\int_0^{10\pi} |\sin x|\,dx = 10\left[\int_0^{\pi/2} \sin x\,dx + \int_{\pi/2}^{\pi} \sin x\,dx\right]$

$= 10\times[\cos x]_0^{\pi/2} + [\cos x]_{\pi/2}^{\pi}$; $10[1+1] = 10\times 2 = 20$

19. $\int_0^{\pi/4} \tan^n x(1+\tan^2 x)dx = \int_0^{\pi/4} \tan^n x \sec^2 x\,dx = \int_0^1 t^n dt$ where $t = \tan x$

$I_n + I_{n+2} = \frac{1}{n+1}; \Rightarrow \lim_{x\to\infty} n[I_n + I_{n+2}] = \lim_{x\to\infty} n.\frac{1}{n+1} = \frac{n}{n+1} = \frac{n}{n\left(1+\frac{1}{n}\right)} = 1$

20. $\int_1^0 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx = 0 + \int_1^{\sqrt{2}} dx = \sqrt{2} - 1$

21. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x} + 2\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x}$

$= 0 + 4\int_0^{\pi} \frac{x\sin x dx}{1+\cos^2 x}$ $I = 4\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}$

$I = 4\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x} \Rightarrow I = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} - 4\pi\int \frac{x\sin x}{1+\cos^2 x} \Rightarrow 2I = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$

put cos x = t and solve it.

22. We have, $\lim_{x\to 2} \frac{xf(2)-2f(x)}{x-2}\left(\frac{0}{0}\right) = \lim_{x\to 2} f(2) - 2f'(x) = f(2) - 2f'(2) = 4 - 2\times 4 = -4$

23. Let $|z| = |\omega| = r$ $\therefore$ $z = re^{i\theta}, \omega = re^{i\phi}$ where $\theta + \phi = \pi$ $\therefore \bar{\omega} = re^{-i\phi}$

$\therefore z = re^{i(\pi-\phi)} = re^{i\pi}.e^{-i\phi} = -re^{-i\phi} = -\bar{\omega}$

24. Given $|z - 4| < |z - 2|$ Let $z = x + iy$
$\Rightarrow |(x-4) + iy)| < |(x-2) + iy| \Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$
$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x \Rightarrow x > 3 \Rightarrow Re(z) > 3$

26. Let a = first term of G.P.
r = common ratio of G.P.; Then G.P. is a, ar, ar^2

Given $s_\infty = 20 \Rightarrow \frac{a}{1-r} = 20 \quad \Rightarrow a = 20(1-r)$ .... (i)

Also $a^2 + a^2r^2 + a^2r^4 + ...to\ \infty = 100 \Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow a^2 = 100(1-r)(1+r)$ .... (ii)

From (i), $a^2 = 400\ (1-r)^2$; From (ii) and (iii), we get $100\ (1- r)(1+r) = 400\ (1-r)^2$
$\Rightarrow 1 + r = 4 - 4r \Rightarrow 5r = 3 \Rightarrow r = 3/5$

27. $1^3 - 2^3 + 3^3 - 4^3 + ... + 9^3$
$= 1^3 + 3^3 + 5^3 + ... + 9^3 - (2^3 + 4^3 + ... + 8^3)$
$= S_1 - S_2$
For S_1, $t_n = (2n - 1)^3 = 8n^3 - 12n^2 + 6n - 1$

$S_1 = \Sigma t_n = 8\Sigma n^3 - 12\Sigma n^2 + 6\Sigma n - \Sigma 1$

$= \frac{8n^2(n+1)^2}{4} - \frac{12n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2} - n$

Here n = 5. Hence $S_1 = 2 \times 25 \times 36 - 2 \times 5 \times 6 \times 11 + 3 \times 30 - 5$
= 1800 - 660 + 90 -5 = 1890 - 665 = 1225

For $S_2, t_n = 8n^3; S_2 = \Sigma t_n = 8\Sigma n^3 = \frac{8n^2(n+1)^2}{4} = 2 \times 16 \times 25 = 800.$ (for n = 4)

$\therefore$ Required sum = 1225 - 800 = 425.

28. Let α, β and y, δ are the roots of the equations

$x^2 + ax + b = 0$ and $x^2 + bx + a = 0 \quad \therefore \alpha + \beta = -a, \alpha\beta = b$ and $y + \delta = -b, y\delta = a$

Given $\alpha - \beta = y - \delta \Rightarrow (\alpha - \beta)^2 = (y - \delta)^2 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (y + \delta)^2 - 4y\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a \Rightarrow (a^2 - b^2) + 4\ (a - b) = 0 \Rightarrow a + b + 4 = 0 \quad (\because a \neq b)$

30. $p + q = - p$ and $pq = q \Rightarrow q(p -1) = 0 \Rightarrow q = 0$ or $p = 1$
If q = 0, then p = 0. i.e. p = q $\quad \therefore$ p = 1 and q = -2

31. $ab + bc + ca = \frac{(a+b+c)^2 - 1}{2} < 1$

32. Required number of numbers = $5 \times 6 \times 6 \times 4 = 36 \times 20 = 720$
33. Required number of numbers = $3 \times 5 \times 5 \times 5 = 375$
34. Required numbers are 5! + 5! - 4! = 216
35. Required sum = (2 + 4 + 6 + ... + 100) + (5 + 10 + 15 + ... + 100) - (10 + 20 + ... + 100)
= 2550 + 1050 - 530 = 3050
36. We have $t_{p+1} = {}^{p+q}C_p\ x^p$ and $t_{q+1} = {}^{p+q}C_q\ x^q \qquad {}^{p+q}C_p = {}^{p+q}C_q$.

37. We have $2^n = 4096 = 2^{12} \Rightarrow n = 12$; So middle term = t_7 ; $t_7 = t_{6+1} = {}^{12}C_6 = \frac{12!}{6!\,6!} = 924$

39. $t_{r+2} = {}^{2n}C_{r+1}\ x^{r+1}$; $\quad t_{3r} = {}^{2n}C_{3r-1}\ x^{3r-1}$
Given ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1} \Rightarrow {}^{2n}C_{2n-(r+1)} = {}^{2n}C_{3r-1} \Rightarrow 2n - r - 1 = 3r - 1 \Rightarrow 2n = 4r$

40. We have $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ By $R_3 \to R_3 - (xR_1 + R_2)$ $= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+x) \end{vmatrix}$

$= (ax^2 + 2bx + c)\ (b^2 - ac) = (+)\ (-) = -ve$

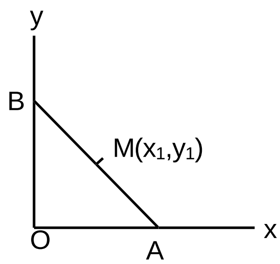
41. $a_1 = \sqrt{7} < 7.$ Let $a_m < 7.$ Then $a_m + 1 = \sqrt{7 + a_m} \Rightarrow a_{m+1}^2 = 7 + a_m < 7 + 7 < 14$

$\Rightarrow a_{m+1} < \sqrt{14} < 7$; So $a_n < 7\ \forall\ n \therefore a_n > 3$

43. Equation of AB is $x\cos\alpha + y\sin\alpha = p \Rightarrow \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1 \Rightarrow \frac{x}{p/\cos\alpha} + \frac{y}{p/\sin\alpha} = 1$

So co-ordinates of A and B are $\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$; So coordinates of mid point of AB are

$\left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right) = (x_1, y_1)$ (let) ; $x_1 = \frac{p}{2\cos\alpha}$ & $y_1 = \frac{p}{2\sin\alpha}$;



$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = p/2y_1$; $\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \frac{p^2}{4}\left(\frac{1}{x_1^2} + \frac{1}{y_1^2}\right) = 1$

Locus of (x_1, y_1) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

45. $3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0 \Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

46. Equation of circles $x^2 + y^2 = 1 = (1)^2$

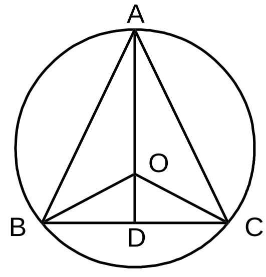
$\Rightarrow x^2 + y^2 = (y - mx)^2 \Rightarrow x^2 = m^2x^2 - 2\,mxy \Rightarrow x^2(1-m^2) + 2\,mxy = 0$

$\tan 45 = \pm\frac{2\sqrt{m^2 - 0}}{1 - m^2} = \frac{\pm 2m}{1 - m^2} \Rightarrow 1 - m^2 = \pm 2m \Rightarrow m^2 \pm 2m - 1 = 0$

$\Rightarrow m = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

47. Let (h, k) be the centre of any such circle. Equation of such circle is $(x - h)^2 + (y - k)^2 = 3^2$. Since (h, k) lies on $x^2 + y^2 = 25$ $\therefore h^2 + k^2 = 25$.
$x^2 + y^2 - (2xh + 2yk) + 25 = 9$; Locus of (h, k) is $x^2 + y^2 = 16$, which clearly satisfies (a).

49. Let ABC be an equilateral triangle, whose median is AD.
Given AD = 3a



In ΔABD, $AB^2 = AD^2 + BD^2$;

$\Rightarrow x^2 = 9a^2 + (x^2/4)$ where $AB = BC = AC = x.$ $\frac{3}{4}x^2 = 9a^2 \Rightarrow x^2 = 12a^2$

In ΔOBD, $OB^2 = OD^2 + BD^2$

$\Rightarrow r^2 = (3a - r)^2 + \frac{x^2}{4} \Rightarrow r^2 = 9a^2 - 6ar + r^2 + 3a^2; \Rightarrow 6ar = 12a^2 \Rightarrow r = 2a$

So equation of circle is $x^2 + y^2 = 4a^2$

50. Any tangent to the parabola $y^2 = 8ax$ is

$y = mx + \frac{2a}{m}$ .... (i)

If (i) is a tangent to the circle, $x^2 + y^2 = 2a^2$ then, $\sqrt{2}a = \pm\frac{2a}{m\sqrt{m^2+1}}$

$\Rightarrow$ $m^2(1+m^2)=2 \Rightarrow (m^2+2)(m^2-1)=0$; $\Rightarrow m = \pm 1$

So from (i), $y = \pm(x+2a)$

51. $r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c} \Rightarrow s-a < s-b < s-c \Rightarrow -a < -b < -c \Rightarrow a > b > c$

52. The given equation is tan x + sec x = 2cosx $\Rightarrow$ sin x + 1 = $2\cos^2 x$

$\Rightarrow$ sinx + 1 = $2(1-\sin^2 x)$ $\Rightarrow$ $2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow$ (2 sin x - 1) (sinx + 1) = 0 $\Rightarrow$ sinx = $\frac{1}{2}$, -1 $\Rightarrow$ x = 30^0, 150^0, 270^0.

54. We have $\lim_{n\to\infty}\frac{1^p+2^p+.....+n^p}{n^{p+1}}$; $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^p}{n^p.n} = \int_0^1 x^p dx = \left[\frac{x^{p+1}}{p+1}\right]_0^1 = \frac{1}{p+1}$

55. Since $\lim_{x\to 0}[x]$ does not exist, hence the required limit does not exist

56. $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ $\left(\frac{0}{0}\right)$ form

Using L' Hospital's rule $\lim_{x\to\infty}\frac{\frac{1}{2\sqrt{f(x)}}f'(x)}{1/2\sqrt{x}} = \frac{f'(1)}{\sqrt{f(1)}} = \frac{2}{1} = 2$

58. $\because f''(x) - g''(x) = 0$

Integrating, $f'(x) - g'(x) = c \Rightarrow f'(1) - g'(1) = c \Rightarrow 4 - 2 = c \Rightarrow c = 2$

$\therefore f'(x) - g'(x) = 2$; Integrating, $f(x) - g(x) = 2x + c_1$

$\Rightarrow f(2) - g(2) = 4 + c_1 \Rightarrow 9 - 3 = 4 + c_1 \Rightarrow c_1 = 2 \quad \therefore f(x) - g(x) = 2x + 2$

At x = 3/2, f(x) - g(x) = 3 + 2 = 5

59. f(x + y) = f(x) $\times$ f(y)

Differentiate with respect to x, treating y as constant

$f'(x+y) = f(x)\, f(y)$

Putting x = 0 and y = x, we get $f'(x) = f'(0)\, f(x)$; $\Rightarrow f'(5) = 3f(5) = 3\times 2 = 6$

60 Distance of origin from (x, y) = $\sqrt{x^2+y^2}$

$= \sqrt{a^2+b^2-2ab\cos\left(t-\frac{at}{b}\right)} = \sqrt{a^2+b^2-2ab}$ $\left[\because \max.\cos\left(t-\frac{at}{b}\right)=1\right] = a - b$

61. Let f(x) = $\frac{ax^3}{3}+\frac{bx^2}{2}+cx \Rightarrow f(0)=0$ and $f(1) = \frac{a}{3}+\frac{b}{2}+c = \frac{2a+3b+6c}{6} = 0$

Also f(x) is continuous and differentiable in [0,1] and [0, 1[. So by Rolle's theorem, $f'(x) = 0$.

i.e. $ax^2 + bx + c = 0$ has at least one root in [0, 1]

62. We have $\int_0^2 f(x)\,dx = \frac{3}{4}$; Now, $\int_0^2 x f'(x) dx = x\int_0^2 f'(x) dx - \int_0^2 f(x)\,dx$

$= [xf(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4} = 0 - \frac{3}{4}\ (\because f(2)=0) = -\frac{3}{4}$

64. We have, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\frac{\pi}{6} = 4\times 2\times\frac{\sqrt{3}}{2} = 4\sqrt{3}$.

Now, $(\vec{a}\times\vec{b})^2 + (\vec{a}.\vec{b})^2 = a^2b^2 \Rightarrow (\vec{a}\times\vec{b})^2 + 48 = 16\times 4 \Rightarrow (\vec{a}\times\vec{b})^2 = 16$

65. We have, $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = (\vec{a}\times\vec{b}).\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\}$

$= (\vec{a}\times\vec{b}).\{(\vec{m}.\vec{a})\,\vec{c} - (\vec{m}.\vec{c})\vec{a}\}$ (where $\vec{m}=\vec{b}\times\vec{c}$)

$= \{(\vec{a}\times\vec{b}).\vec{c}\}.\{\vec{a}.(\vec{b}\times\vec{c})\} = [\vec{a}\vec{b}\vec{c}]^2 = 4^2 = 16$

66. $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{b}+\vec{c} = -\vec{a} \Rightarrow (\vec{b}+\vec{c})^2 = (\vec{a})^2 = 5^2+3^2+2\vec{b}\vec{c} = 7^2$

$\Rightarrow 2|\vec{b}||\vec{c}|\cos\theta = 49-34 = 15 \Rightarrow 2\times5\times3\cos\theta = 15 \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \frac{\pi}{3} = 60^0$

67. We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow (\vec{a}+\vec{b}+\vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0 \Rightarrow 25+16+9+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$\Rightarrow (\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = -25 \quad \therefore |\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}| = 25$

69. We have $\vec{a}\times\vec{b} = 39\vec{k} = \vec{c}$

Also $|\vec{a}| = \sqrt{34}, |\vec{b}| = \sqrt{45}, |\vec{c}| = 39 \therefore \quad |\vec{a}|:|\vec{b}|:|\vec{c}| = \sqrt{34}:\sqrt{45}:39$

71. $P(A\cup B) = P(A)+P(B)-P(A\cap B) \Rightarrow \frac{3}{4} = 1-P(\bar{A})+P(B)-\frac{1}{4}$

$\Rightarrow 1 = 1-\frac{2}{3}+P(B) \Rightarrow P(B) = \frac{2}{3};\quad \text{Now}, P(\bar{A}\cap B) = P(B)-P(A\cap B) = \frac{2}{3}-\frac{1}{4} = \frac{5}{12}$

72. The event follows binomial distribution with n = 5, p = 3/6 = 1/2
q = 1 - p = 1/2 $\therefore$ Variance npq = 5/4

73. Equation of plane through (1, 0, 0) is
a(x - 1) + by + cz = 0 .... (i)
(i) passes through (0, 1, 0)

$-a+b=0 \Rightarrow b = a$; Also, $\cos45^0 = \frac{a+a}{\sqrt{2(2a^2+c^2)}}$

$\Rightarrow 2a = \sqrt{2a^2+c^2} \Rightarrow 2a^2 = c^2 \Rightarrow c = \sqrt{2}a$

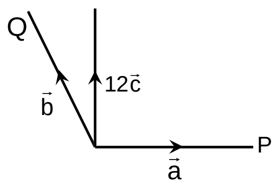
So. d.r. of normal are a, a $\sqrt{2}a$ i.e. $1, 1, \sqrt{2}$

74. Let two forces be P and Q. Given P + Q = 18 and $P\hat{a}+Q\hat{b} = 12\hat{c} \Rightarrow P\hat{a}-12\hat{c} = \overline{Q}\hat{b}$

$\Rightarrow P^2+144 = Q^2 = (18-P)^2; \Rightarrow P^2+144 = 324-36P+P^2$

$\Rightarrow 36P = 180 \Rightarrow P = 5$ and $Q = 13$

(where $\vec{a}$ and $\vec{b}$ are unit vectors along P and Q).



KEY FOR AIEEE - 2002 PAPER

PHYSICS &		40.	c	81.	b	122.	d	12.	a	53.	b
CHEMISTRY		41.	a	82.	b	123.	d	13.	b	54.	a
1.	a	42.	b	83.	a	124.	d	14.	d	55.	d
2.	b	43.	a	84.	a	125.	d	15.	a	56.	a
3.	b	44.	c	85.	a	126.	d	16.	b	57.	b
4.	b	45.	a	86.	a	127.	a	17.	b	58.	d
5.	c	46.	d	87.	d	128.	c	18.	d	59.	c
6.	c	47.	b	88.	a	129.	b	19.	b	60.	a
7.	a	48.	b	89.	a	130.	c	20.	c	61.	a
8.	a	49.	b	90.	b	131.	d	21.	b	62.	d
9.	b	50.	d	91.	b	132.	a	22.	c	63.	a
10.	c	51.	b	92.	b	133.	a	23.	b	64.	b
11.	c	52.	c	93.	a	134.	b	24.	c	65.	a
12.	b	53.	b	94.	c	135.	d	25.	b	66.	a
13.	b	54.	d	95.	a	136.	d	26.	c	67.	a
14.	a	55.	a	96.	a	137.	a	27.	a	68.	b
15.	a	56.	d	97.	c	138.	c	28.	a	69.	b
16.	c	57.	b	98.	c	139.	a	29.	c	70.	c
17.	c	58.	c	99.	a	140.	d	30.	a	71.	a
18.	b	59.	b	100.	c	141.	d	31.	a	72.	d
19.	b	60.	a	101.	a	142.	b	32.	d	73.	b
20.	b	61.	b	102.	a	143.	c	33.	c	74.	a
21.	c	62.	d	103.	d	144.	b	34.	d	75.	a
22.	b	63.	c	104.	c	145.	b	35.	b		
23.	b	64.	d	105.	d	146.	d	36.	a		
24.	c	65.	a	106.	b	147.	a	37.	c		
25.	a	66.	b	107.	a	148.	c	38.	c		
26.	c	67.	a	108.	c	149.	d	39.	c		
27.	a	68.	b	109.	c	150.	c	40.	c		
28.	c	69.	c	110.	c	**MATHEMATICS**		41.	b		
29.	a	70.	d	111.	c	1.	a	42.	a		
30.	d	71.	a	112.	b	2.	a	43.	d		
31.	b	72.	a	113.	c	3.	c	44.	a		
32.	a	73.	c	114.	b	4.	a	45.	a		
33.	c	74.	a	115.	c	5.	b	46.	c		
34	b	75.	c	116.	c	6.	d	47.	a		
35.	a	76.	c	117.	a	7.	a	48.	b		
36.	d	77.	c	118.	a	8.	a	49.	c		
37.	c	78.	b	119.	d	9.	b	50.	b		
38.	b	79.	a	120.	a	10.	a	51.	a		
39.	a	80.	b	121.	b	11.	c	52.	b		