FIITJEE Solutions to

IITJEE-2007

(Paper-I, Code-7)

Time: 3 hours ***M. Marks: 243***

Note: ***(i)*** The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.

(ii) **Section I** contains 9 multiple choice questions which have only one correct answer. Each question carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iii) **Section II** contains 4 questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason).
Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1
Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1
Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iv) **Section III** contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has only one correct answer and carries ***+4 marks*** for correct answer and ***– 1 mark*** for wrong answer.

(v) **Section IV** contains 3 questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column and each question carries ***+6 marks*** and marks will be awarded if all the four parts are correctly matched. No marks will be given for any wrong match in any question. There is no negative marking.

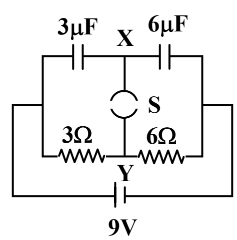
PART- I (PHYSICS)

SECTION – I

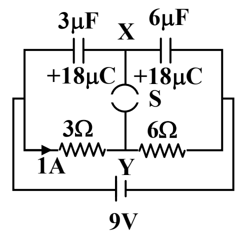
Straight Objective Type

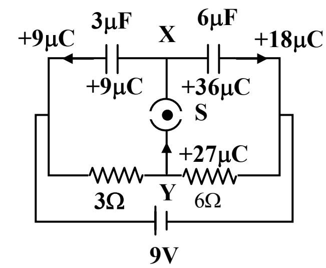
This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

1. A circuit is connected as shown in the figure with the switch S open. When the switch is closed the total amount of charge that flows from Y to X is
(A) 0 (B) 54μC
(C) 27 μC (D) 81 μC



Sol. **(C)**
27 μC





Initial charge distribution (when switch S is open) Final charge distribution (when switch S is closed)

2. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.
(A) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
(B) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
(C) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
(D) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

Sol. **(A)**
$dV = -\vec{E}\cdot d\vec{r}$
and $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$
where r is distance from the axis of cylindrical charge distribution (r is equal to or greater than radius of cylindrical charge distribution).

3. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is
(A) $E\left({}^{236}_{92}U\right) > E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(B) $E\left({}^{236}_{92}U\right) < E\left({}^{137}_{53}I\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(C) $E\left({}^{236}_{92}U\right) < E\left({}^{140}_{56}Ba\right) + E\left({}^{94}_{36}Kr\right) + 2E(n)$
(D) $E\left({}^{236}_{92}U\right) = E\left({}^{140}_{56}Ba\right) + E\left({}^{94}_{36}Kr\right) + 2E(n)$

3. **(A)**
Rest mass energy of U will be greater than the rest mass energy of the nucleus in which it breaks (as conservation of momentum is always followed)

4. In an experiment to determine the focal length (f) of a concave mirror by the u–v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then,
(A) $x < f$
(B) $f < x < 2f$
(C) $x = 2f$
(D) $x > 2f$

Sol. **(B)**
Due to parallax

5. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is
(A) 802 nm
(B) 823 nm
(C) 1882 nm
(D) 1648 nm

Sol. **(B)**
Transition from ∞ to n = 3 will produce smallest wavelength in infrared region.

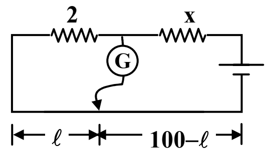
6. A resistance of 2 Ω is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2Ω, is connected across the other gap. When these resistance are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is
(A) 3 Ω
(B) 4 Ω
(C) 5 Ω
(D) 6 Ω

Sol. **(A)**

$$\frac{2}{x} = \frac{\ell}{100-\ell} \quad \ldots(i)$$

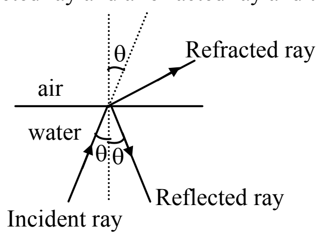
$$\frac{x}{2} = \frac{\ell+20}{80-\ell} \quad \ldots(ii)$$

Solving (i) and (ii) x = 3Ω

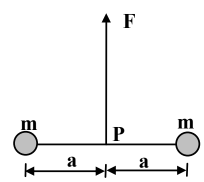


7. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be
(A) only a reflected ray and no refracted ray
(B) only a refracted ray and no reflected ray
(C) a reflected ray and a refracted ray and the angle between them would be less than 180° – 2θ
(D) a reflected ray and a refracted ray and the angle between them would be greater than 180° – 2θ

Sol. **(C)**



8. Two particle of mass m each are tied at the ends of a light string of length 2a. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the center P (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes 2x is



(A) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (B) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(C) $\frac{F}{2m}\frac{x}{a}$ (D) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

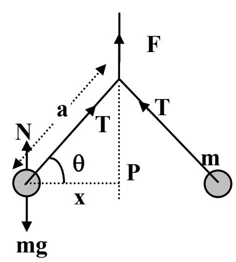
Sol. **(B)**

2T sin θ = F

T cos θ = mA

$$2\tan\theta = \frac{F}{mA}$$

$$A = \frac{F}{2m}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$



9. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,
(A) negative and distributed uniformly over the surface of the sphere
(B) negative and appears only at the point on the sphere closest to the point charge
(C) negative and distributed non-uniformly over the entire surface of the sphere
(D) zero

Sol. **(D)**

SECTION – II

Assertion - Reason Type

This section contains 4 questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

10. **STATEMENT-1**
The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

because

STATEMENT-2
Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.
(A) Statement-1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(C)**

11. **STATEMENT-1**
If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.

because

STATEMENT -2
When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.
(A) Statement-1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

12. **STATEMENT-1**
A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30^0 with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

because

STATEMENT-2
The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.
(A) Statement -1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(C)**

13. **STATEMENT-1**
In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.

because

STATEMENT-2
In an elastic collision, the linear momentum of the system is conserved.
(A) Statement -1 is True, Statement-2 is True; Statement -2 is a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

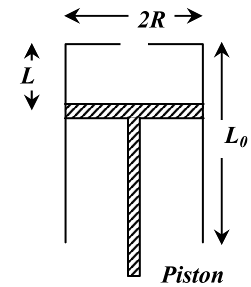
SECTION – III

Linked Comprehension Type

*This section contains 2 paragraphs P_{14-16} and P_{17-19}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

P_{14-16} : Paragraph for Question Nos. 14 to 16

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



14. The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be

(A) P_0 (B) $\frac{P_0}{2}$

(C) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$

Sol. **(A)**

15. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(A) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$ (B) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

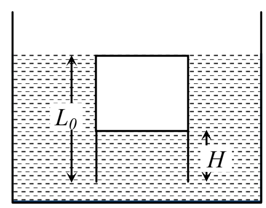
Sol. **(D)**

$Mg + P(\pi R^2) = P_0\pi R^2$

$P_0(2L\pi R^2) = P(x\pi R^2)$ ($P_1 \cdot V_1 = P_2V_2$ for isothermal process)

$x = \left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

16. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(A) $\rho g(L_0-H)^2 + P_0(L_0 - H) + L_0P_0 = 0$
(B) $\rho g(L_0-H)^2 - P_0(L_0 - H) - L_0P_0 = 0$
(C) $\rho g(L_0-H)^2 + P_0(L_0 - H) - L_0P_0 = 0$
(D) $\rho g(L_0-H)^2 - P_0(L_0 - H) + L_0P_0 = 0$

Sol. **(C)**

$\pi R^2 P_0 L_0 = P(L_0 - H)\pi R^2$ . . . (i)

$P = P_0 + \rho g(L_0 - H)$ . . . (ii)

Solving (i) & (ii), we get the answer.

P_{17-19} : Paragraph for Question Nos. 17 to 19

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

17. The ratio of x_1/x_2 is

(A) 2 (B) $\frac{1}{2}$

(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

Sol. **(C)**

$\frac{1}{2}kx_1^2 = \frac{1}{2}I(2\omega)^2$

$$\frac{1}{2}kx_2^2 = \frac{1}{2}(2I)(\omega)^2$$

$$\frac{x_1}{x_2} = \sqrt{2}$$

18. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$

(C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

Sol. **(A)**

Applying conservation of angular momentum

$$\omega' = \frac{I(2\omega) + 2I(\omega)}{3I} = \frac{4\omega}{3} \qquad \text{. . . (i)}$$

$$\omega' = \omega + \frac{\tau}{2I}t \qquad \text{. . . (ii)}$$

From (1) & (ii), $\tau = \frac{2I\omega}{3t}$

19. The loss of kinetic energy during the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$

(C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

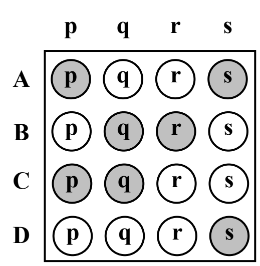
Sol. **(B)**

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct match are A-p, A-s, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



20. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II.** Match the physical quantities in **Column I** with the units in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) GM_eM_s $G \rightarrow$ universal gravitational constant, $M_e \rightarrow$ mass of the earth, $M_s \rightarrow$ mass of the Sun	(p) (volt) (coulomb) (metre)

(B) $\frac{3RT}{M}$; R $\rightarrow$ universal gas constant, T $\rightarrow$ absolute temperature, M $\rightarrow$ molar mass	(q) (kilogram) $(\text{metre})^3$ $(\text{second})^{-2}$
(C) $\frac{F^2}{q^2B^2}$; F $\rightarrow$ force, q $\rightarrow$ charge, B $\rightarrow$ magnetic field	(r) $(\text{meter})^2$ $(\text{second})^{-2}$
(D) $\frac{GM_e}{R_e}$, G $\rightarrow$ universal gravitational constant, M_e $\rightarrow$ mass of the earth, R_e $\rightarrow$ radius of the earth	(s) (farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

Sol. A $\rightarrow$(p) & (q), B $\rightarrow$(r) & (s), C $\rightarrow$(r) & (s), D $\rightarrow$(r) & (s)

21. Some laws/processes are given in **Column I**. Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) Transition between two atomic energy levels	(p) Characteristic X-rays
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

Sol. A $\rightarrow$(p) & (r), B $\rightarrow$(q) & (s), C $\rightarrow$(p), D $\rightarrow$(q)

22. **(C)olumn I** gives certain situations in which a straight metallic wire of resistance R is used and **Column II** gives some resulting effects. Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) A charged capacitor is connected to the ends of the wire	(p) A constant current flows through the wire
(B) The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	(q) Thermal energy is generated in the wire
(C) The wire is placed in a constant electric field that has a direction along the length of the wire.	(r) A constant potential difference develops between the ends of the wire
(D) A battery of constant emf is connected to the ends of the wire	(s) Charges of constant magnitude appear at the ends of the wire

Sol. A $\rightarrow$(q), B $\rightarrow$(r) & (s), C $\rightarrow$(r) & (s), D $\rightarrow$(p), (q) & (r)

PART- II (CHEMISTRY)

SECTION – I

Straight Objective Type

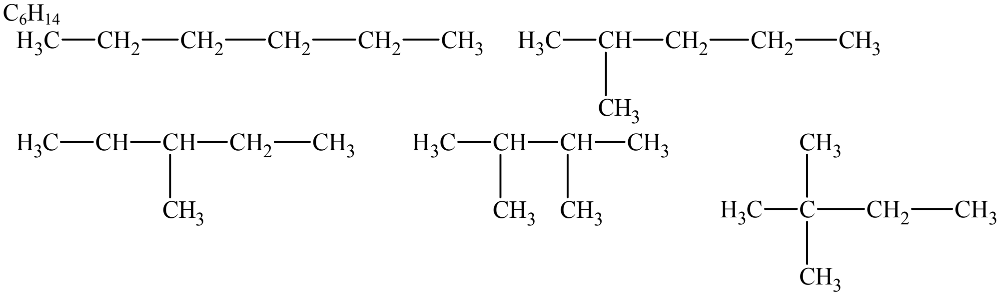
*This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

23. The number of structural isomers for C_6H_{14} is

(A) 3 (B) 4
(C) 5 (D) 6

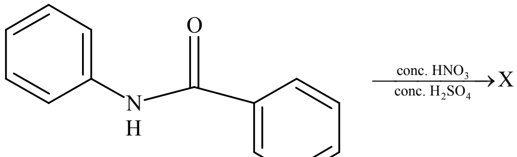
Sol. **(C)**

C_6H_{14}



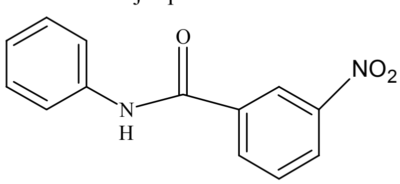
Hence (C) is correct.

24. In the following reaction,

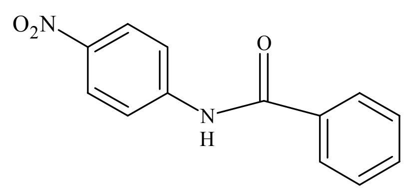


the structure of the major product **'X'** is

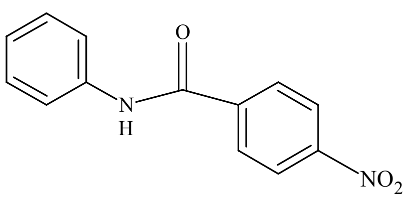
(A)



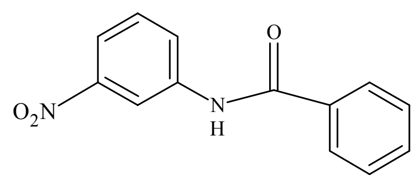
(B)



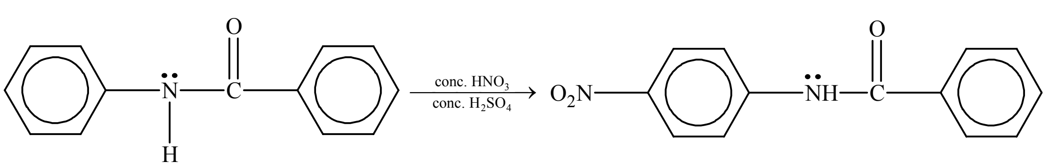
(C)



(D)



Sol. **(B)**



Due to presence of lone pair of electron on nitrogen atom, it will activate the ring and it will stabilize intermediate cation at o and p positions.

Hence (B) is correct.

25. When 20 g of naphthoic acid ($C_{11}H_8O_2$) is dissolved in 50 g of benzene (K_f = 1.72 K kg mol^{-1}), a freezing point depression of 2 K is observed. The van't Hoff factor (i) is

(A) 0.5 (B) 1
(C) 2 (D) 3

Sol. **(A)**

$\Delta T_f = K_f \times \text{molality} \times i$

$2 = 1.72 \times \frac{20}{172} \times \frac{1000}{50} \times i$

$i = 0.5$

Hence (A) is correct.

26. Among the following, the paramagnetic compound is

(A) Na_2O_2 (B) O_3
(C) N_2O (D) KO_2

Sol. **(D)**

$O_2^{2-} = \sigma 1s^2\, \sigma^* 1s^2, \sigma 2s^2\, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^2$

Number of unpaired electrons = 0.

$N = N \longrightarrow O$ Number of unpaired electrons = 0

$O{=}\overset{\oplus}{O}{-}\overset{\ominus}{O} \longleftrightarrow O{=}O \longrightarrow O$ Number of unpaired electrons = 0

$O_2^{-} = \sigma 1s^2, \sigma^* 1s^2 \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^1$

Number of unpaired electrons = 1

Thus O_2^- is paramagnetic.

Hence (D) is correct.

27. The value of $\log_{10}K$ for a reaction $A \rightleftharpoons B$ is

(Given : $\Delta_r H^\circ_{298K} = -54.07 \text{kJ mol}^{-1}$, $\Delta_r S^\circ_{298K} = 10 \text{JK}^{-1}\text{mol}^{-1}$ and R= 8.314 JK^{-1}mol^{-1}; 2.303×8.314×298 = 5705)

(A) 5 (B) 10
(C) 95 (D) 100

Sol. **(B)**

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = -54.07 \times 1000 - 298 \times 10 = -57050 \text{ J mol}^{-1}$

$-57050 = -5705 \log_{10}K$

$\log_{10}K = 10$

Hence (B) is correct

28. The species having bond order different from that in CO is

(A) NO^- (B) NO^+
(C) CN^- (D) N_2

Sol. **(A)**

NO^- (16 electron system)

Bond order = 2.

$NO^{\oplus}$, CN^- and N_2 are isoelectronic with CO therefore all have same bond order (= 3)

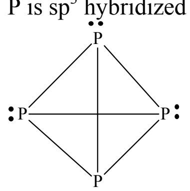
Hence (A) is correct.

29. The percentage of p-character in the orbitals forming P–P bonds in P_4 is

(A) 25 (B) 33
(C) 50 (D) 75

Sol. **(D)**

P is sp^3 hybridized in P_4.

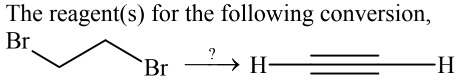


Hence (D) is correct.

30. Extraction of zinc from zinc blende is achieved by
(A) electrolytic reduction
(B) roasting followed by reduction with carbon
(C) roasting followed by reduction with another metal
(D) roasting followed by self-reduction

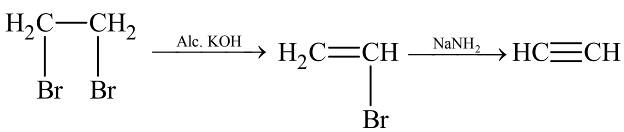
Sol. **(B)**
Option (B) is correct.

31. The reagent(s) for the following conversion,



is/are
(A) alcoholic KOH
(B) alcoholic KOH followed by $NaNH_2$
(C) aqueous KOH followed by $NaNH_2$
(D) Zn/CH_3OH

Sol. **(B)**



Because $CH_2 = CH - Br$ has partial C – Br double bond character, it requires more stronger base to remove HBr.
Hence(B) is correct.

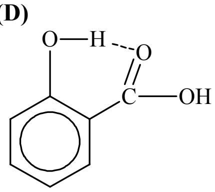
SECTION – II

Assertion-Reason Type

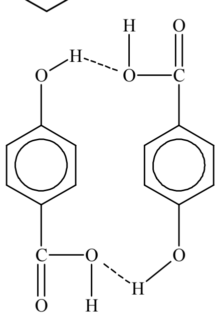
This section contains 4 questions numbered 32 to 35. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

32. **STATEMENT-1**: p-Hydroxybenzoic acid has a lower boiling point than o-hydroxybenzoic acid.
because
STATEMENT-2: o-Hydroxybenzoic acid has intramolecular hydrogen bonding.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement -2 is False.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(D)**



More stabilized by intramolecular hydrogen bonding



More stronger intermolecular forces increases the boiling point.

Hence (D) is correct.

33. STATEMENT-1: Micelles are formed by surfactant molecules above the critical micellar concentration (CMC).
because
STATEMENT-2: The conductivity of a solution having surfactant molecules decreases sharply at the CMC.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(B)**
The formation of micelles takes places only above a particular temperature called Kraft temperature (T_k) and above a particular concentration called critical micelle concentration (CMC).
Each micelle contains at least 100 molecules. Therefore conductivity of the solution decreases sharply at the CMC.
Hence (B) is correct.

34. **STATEMENT-1**: Boron always forms covalent bond.
because
STATEMENT-2: The small size of B^{3+} favours formation of covalent bond.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is True.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(A)**
According to Fajan's rule small cations having high charge density always have tendency to form covalent bond.
Hence (A) is correct

35. **STATEMENT-1**: In water, orthoboric acid behaves as a weak monobasic acid.
because
STATEMENT-2: In water, orthoboric acid acts as a proton donor.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

Sol. **(C)**
H_3BO_3 (orthoboric acid) is a weak lewis acid.

$$H_3BO_3 + H_2O \rightleftharpoons B(OH)_4^- + H^{\oplus}$$

It does not donate proton rather it acceptors OH^- form water.
Hence (C) is correct

SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs $C_{36\text{-}38}$ and $C_{39\text{-}41}$. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

$C_{36\text{-}38}$: Paragraph for question Nos 36 to 38

Chemical reactions involve interaction of atoms and molecules. A large number of atoms/molecules (approximately 6.023×10^{23}) are present in a few grams of any chemical compound varying with their atomic/molecular masses. To handle such large numbers conveniently, the mole concept was introduced. This concept has implications in diverse areas such as analytical chemistry, biochemistry, electrochemistry and radiochemistry. The following example illustrates a typical case, involving chemical/electrochemical reaction, which requires a clear understanding of the mole concept.

A 4.0 molar aqueous solution of NaCl is prepared and 500 mL of this solution is electrolysed. This leads to the evolution of chlorine gas at one of the electrodes (atomic mass: Na = 23, Hg = 200; 1 Faraday=96500 coulombs)

36. The total number of moles of chlorine gas evolved is
(A) 0.5 (B) 1.0
(C) 2.0 (D) 3.0

Sol. **(B)**

$NaCl \longrightarrow Na^+ + Cl^-$

At anode:

$2Cl^- \longrightarrow Cl_2$

Moles of Cl^- = 2 in 500 ml.
Therefore 1 mole of Cl_2 evolves.
Hence (B) is correct.

37. If the cathode is a Hg electrode, the maximum weight (g) of amalgam formed from this solution is
(A) 200 (B) 225
(C) 400 (D) 446

Sol. **(D)**
Na – Hg (amalgam) formed = 2 moles at cathode.
Hence (D) is correct.

38. The total charge (coulombs) required for complete electrolysis is
(A) 24125 (B) 48250
(C) 96500 (D) 193000

Sol. **(D)**
2 moles of electrons (2 Faraday) are required.
1F = 96500
2F = 193000
Hence (D) is correct.

$C_{39\text{-}41}$: Paragraph for Question Nos. 39 to 41

The noble gases have closed-shell electronic configuration and are monoatomic gases under normal conditions. The low boiling points of the lighter noble gases are due to weak dispersion forces between the atoms and the absence of other interatomic interactions.

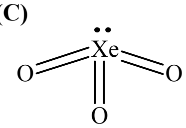
The direct reaction of xenon with fluorine leads to a series of compounds with oxidation numbers +2, +4 and +6. XeF_4 reacts violently with water to give XeO_3. The compounds of xenon exhibit rich stereochemistry and their geometries can be deduced considering the total number of electron pairs in the valence shell.

39. Argon is used in arc welding because of its
(A) low reactivity with metal (B) ability to lower the melting point of metal
(C) flammability (D) high calorific value

Sol. **(A)**
Argon is used mainly to provide an inert atmosphere in high temperature metallurgical (arc welding of metals/alloys) extraction.
Hence (A) is correct.

40. The structure of XeO_3 is
(A) linear (B) planar
(C) pyramidal (D) T-shaped

Sol. **(C)**



sp^3 hybridized pyramidal structure.

Hence (C) is correct.

41. XeF_4 and XeF_6 are expected to be
(A) oxidizing (B) reducing
(C) unreactive (D) strongly basic

Sol. **(A)**

$6XeF_4 + 12H_2O \longrightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$

$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$

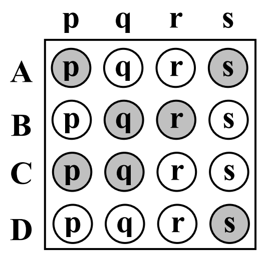
Hence (A) is correct.

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s) in Column-II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:

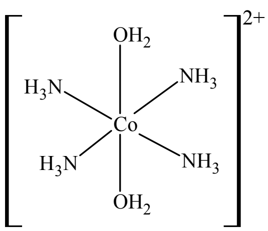


42. Match the complexes in Column-I with their properties listed in Column-II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

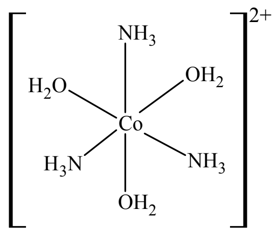
Column-I	Column-II
(A) $[Co(NH_3)_4(H_2O)_2]Cl_2$	(p) geometrical isomers
(B) $[Pt(NH_3)_2Cl_2]$	(q) paramagnetic
(C) $[Co(H_2O)_5Cl]Cl$	(r) diamagnetic
(D) $[Ni(H_2O)_6]Cl_2$	(s) metal ion with +2 oxidation state

Sol. **A → p, q, s B → p, r, s C → q, s D → q, s**

(A)



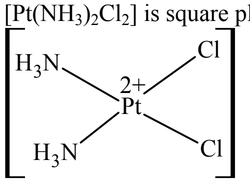
(trans)



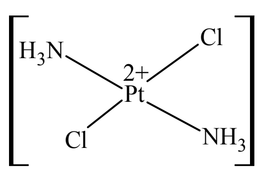
(cis)

$Co^{2+} = 3d^7$ (Paramagnetic)

(B) $[Pt(NH_3)_2Cl_2]$ is square planar.



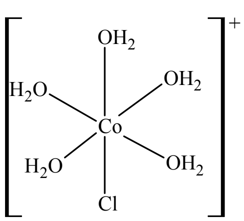
(cis)



(trans)

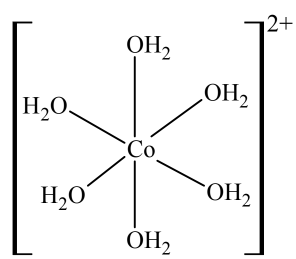
$Pt^{2+} = 5d^8 4s^0$ (diamagnetic)

(C)



$Co^{2+} = 3d^7$ (paramagnetic)

(D)



$Ni^{2+} = 3d^8$ (weak field ligand, paramagnetic)

43. Match gases under specified conditions listed in **Column-I** with their properties/laws in **Column-II.** Indicate your answer by darkening the appropriate bubsbles of the 4 × 4 matrix given in the ORS.

Column-I		**Column-II**	
(A)	hydrogen gas (P = 200 atm, T = 273K)	(p)	compressibility factor ≠ 1
(B)	hydrogen gas (P ~ 0, T = 273K)	(q)	attractive forces are dominant
(C)	CO_2 (P = 1 atm, T = 273K)	(r)	PV = nRT
(D)	real gas with very large molar volume	(s)	P(V – nb) = nRT

Sol. **A → p, s B → r C → p, q D → p, s**

(A) $Z = \frac{PV_m}{RT}$ at high pressure and low temperature.

Equation $\left(P + \frac{an^2}{V^2}\right)(V - nb)$ = nRT reduces to P(V – nb) = nRT.

(B) For hydrogen gas value of Z = 1 at P = 0 and it increase continuously on increasing pressure.

(C) CO_2 molecules have larger attractive forces, under normal conditions.

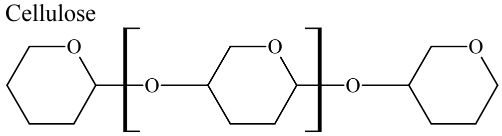
(D) $Z = \frac{PV_m}{RT}$, at very large molar volume Z ≠ 1.

44. Match the chemical substances in Column-I with type of polymers/type of bonds in Column-II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column-I		**Column-II**	
(A)	cellulose	(p)	natural polymer
(B)	nylon-6, 6	(q)	synthetic polymer
(C)	protein	(r)	amide linkage
(D)	sucrose	(s)	glycoside linkage

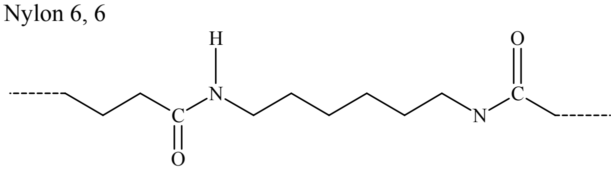
Sol. **A → p, s, B → q, r; C → p, r D → s**

(A) Cellulose



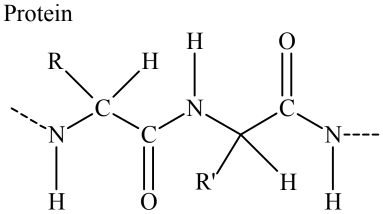
(Glycoside linkage)

(B) Nylon 6, 6



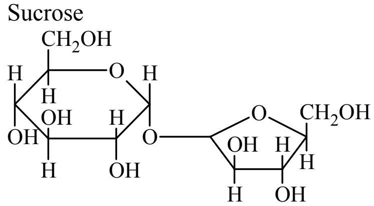
(Amide linakge)

(C) Protein



(Amide linkage)

(D) Sucrose



(Glycoside linkage)

PART III (MATHEMATICS)

SECTION –I

Straight Objective Type

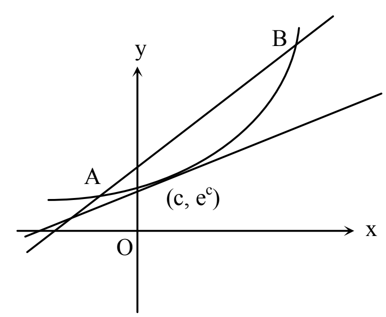
*This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

45. A hyperbola, having the transverse axis of length 2 sinθ, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is
(A) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ (B) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$
(C) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ (D) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Sol. **(A)**
The given ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$
$\Rightarrow a = 2, b = \sqrt{3} \Rightarrow 3 = 4(1 - e^2) \Rightarrow e = \frac{1}{2}$
so that ae = 1
Hence the eccentricity e_1, of the hyperbola is given by
$1 = e_1 \sin\theta \Rightarrow e_1 = \operatorname{cosec}\theta$
$\Rightarrow b^2 = \sin^2\theta(\operatorname{cosec}^2\theta - 1) = \cos^2\theta$
Hence the hyperbola is $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$ or $x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$

46. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$
(A) on the left of x = c (B) on the right of x = c
(C) at no point (D) at all points

Sol. **(A)**
Slope of the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$ is equal to $\frac{e^{c+1} - e^{c-1}}{2} > e^c$
$\Rightarrow$ tangent to the curve $y = e^x$ will intersect the given line to the left of the line x = c.



Alternative
The equation of the tangent to the curve $y = e^x$ at (c, e^c) is
$y - e^c = e^c(x - c)$ …(1)
Equation of the line joining the given points is
$y - e^{c-1} = \frac{e^c(e - e^{-1})}{2}[x - (c-1)]$ …(2)
Eliminating y from (1) and (2), we get
$[x - (c - 1)][2 - (e - e^{-1})] = 2e^{-1}$
or $x - c = \frac{e + e^{-1} - 2}{2 - (e - e^{-1})} < 0 \Rightarrow x < c.$
$\Rightarrow$ the line (1) and (2) meet on the left of the line x = c.

47. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is
(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)\, e^{i\pi/4}$
(C) $(4 + 3i)\, e^{i\pi/4}$ (D) $(3 + 4i)\, e^{i\pi/4}$

Sol. **(D)**

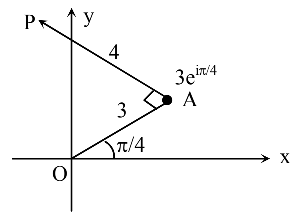
Let OA = 3, so that the complex number associated with A is $3e^{i\pi/4}$.

If z is the complex number associated with P,

then $\frac{z - 3e^{i\pi/4}}{0 - 3e^{i\pi/4}} = \frac{4}{3}e^{-i\pi/2} = -\frac{4i}{3}$

$\Rightarrow 3z - 9e^{i\pi/4} = 12ie^{i\pi/4}$

$\Rightarrow z = (3 + 4i)e^{i\pi/4}$.



48. Let f(x) be differentiable on the interval $(0, \infty)$ such that f(1) = 1, and

$$\lim_{t\to x}\frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

for each x > 0. Then f(x) is

(A) $\frac{1}{3x} + \frac{2x^2}{3}$ (B) $-\frac{1}{3x} + \frac{4x^2}{3}$

(C) $-\frac{1}{x} + \frac{2}{x^2}$ (D) $\frac{1}{x}$

Sol. **(A)**

$\lim_{t\to x}\frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

$\Rightarrow x^2 f'(x) - 2xf(x) + 1 = 0$

$\Rightarrow f(x) = cx^2 + \frac{1}{3x}$ also f(1) = 1

$\Rightarrow c = \frac{2}{3}$.

Hence $f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$.

49. The number of solutions of the pair of equations

$2\sin^2\theta - \cos 2\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$

in the interval $[0, 2\pi]$ is

(A) zero (B) one

(C) two (D) four

Sol. **(C)**

$2\sin^2\theta - \cos 2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{4}$

also $2\cos^2\theta = 3\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow$ two solutions in $[0, 2\pi]$.

50. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

(A) $\frac{2}{9}(p - q)(2q - p)$ (B) $\frac{2}{9}(q - p)(2p - q)$

(C) $\frac{2}{9}(q - 2p)(2q - p)$ (D) $\frac{2}{9}(2p - q)(2q - p)$

Sol. **(D)**

The equation $x^2 - px + r = 0$ has roots (α, β) and the equation

$x^2 - qx + r = 0$ has roots $\left(\frac{\alpha}{2}, 2\beta\right)$.

$\Rightarrow r = \alpha\beta$ and $\alpha + \beta = p$ and $\frac{\alpha}{2} + 2\beta = q$

$\Rightarrow \beta = \frac{2q - p}{3}$ and $\alpha = \frac{2(2p - q)}{3}$

$\Rightarrow \alpha\beta = r = \frac{2}{9}(2q - p)(2p - q)$.

51. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) zero
(B) one
(C) two
(D) three

Sol. **(C)**

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0 \Rightarrow \lambda^6 - 3\lambda^2 - 2 = 0$$

$\Rightarrow (1 + \lambda^2)^2 (\lambda^2 - 2) = 0 \Rightarrow \lambda = \pm\sqrt{2}$.

52. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(A) 1/2
(B) 1/3
(C) 2/5
(D) 1/5

Sol. **(C)**

Let E = event when each American man is seated adjacent to his wife

A = event when Indian man is seated adjacent to his wife

Now, $n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife

Again $n(E) = (5!) \times (2!)^4$

$$\Rightarrow P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)} = \frac{(4!) \times (2!)^5}{(5!) \times (2!)^4} = \frac{2}{5}.$$

Alternative

Fixing four American couples and one Indian man in between any two couples; we have 5 different ways in which his wife can be seated, of which 2 cases are favorable.

$\therefore$ required probability $= \frac{2}{5}$.

53. $$\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\, dt}{x^2 - \frac{\pi^2}{16}}$$ equals

(A) $\frac{8}{\pi} f(2)$
(B) $\frac{2}{\pi} f(2)$
(C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$
(D) $4f(2)$

Sol. **(A)**

$$\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}} \qquad \left(\frac{0}{0}\text{ form}\right)$$

Let $L = \lim_{x\to\frac{\pi}{4}} \frac{f(\sec^2 x)2\sec x \sec x \tan x}{2x}$

$\therefore\ L = \frac{2f(2)}{\pi/4} = \frac{8f(2)}{\pi}$.

SECTION –II

Assertion – Reason Type

*This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT – 1 (Assertion) and STATEMENT -2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.*

---

54. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

STATEMENT -1 : $\overrightarrow{PQ}\times\left(\overrightarrow{RS}+\overrightarrow{ST}\right)\neq\vec{0}$.

because

STATEMENT -2 : $\overrightarrow{PQ}\times\overrightarrow{RS}=\vec{0}$ and $\overrightarrow{PQ}\times\overrightarrow{ST}\neq\vec{0}$.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(C)**

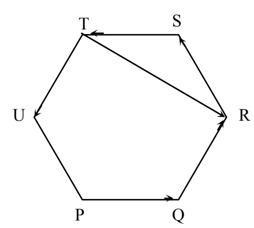
Since $\overrightarrow{PQ} \nparallel \overrightarrow{TR}$ $\quad\because\quad$ $\overrightarrow{TR}$ is resultant of $\overrightarrow{SR}$ and $\overrightarrow{ST}$ vector.

$\Rightarrow\ \overrightarrow{PQ}\times\left(\overrightarrow{RS}+\overrightarrow{ST}\right)\neq 0$.

But for statement 2, we have $\overrightarrow{PQ}\times\overrightarrow{RS}=\vec{0}$

which is not possible as $\overrightarrow{PQ} \nparallel \overrightarrow{RS}$.

Hence, statement 1 is true and statement 2 is false.



55. Let F(x) be an indefinite integral of $\sin^2 x$.

STATEMENT -1 : The function F(x) satisfies $F(x + \pi) = F(x)$ for all real x.

because

STATEMENT -2 : $\sin^2(x + \pi) = \sin^2 x$ for all real x.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

$F(x) = \int \sin^2 x dx = \int \frac{1-\cos 2x}{2} dx$

$\Rightarrow F(x) = \frac{1}{4}(2x - \sin 2x) + c.$

Since, $F(x + \pi) \neq F(x)$.
Hence statement 1 is false.
But statement 2 is true as $\sin^2 x$ is periodic with period π.

56. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

STATEMENT -1 : $P(H_i \mid E) > P(E \mid H_i) \, . \, P(H_i)$ for $i = 1, 2, \ldots, n$

because

STATEMENT -2 : $\sum_{i=1}^{n} P(H_i) = 1$

(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**

Statement : 1

If $P(H_i \cap E) = 0$ for some i, then $P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$

If $P(H_i \cap E) \neq 0$ for $\forall$ i = 1, 2 … n, then

$$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$$

$$= \frac{P\left(\frac{E}{H_i}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right) \cdot P(H_i) \qquad [\text{as } 0 < P(E) < 1]$$

Hence statement 1 may not always be true.
Statement : 2
Clearly $H_1 \cup H_2 \ldots \cup H_n = S$ (sample space)
$\Rightarrow P(H_1) + P(H_2) + \ldots + P(H_n) = 1$.

57. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

STATEMENT -1 : The tangents are mutually perpendicular.

because

STATEMENT -2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$

(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is true; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(A)**
Since the tangents are perpendicular $\Rightarrow$ locus of perpendicular tangents to circle $x^2 + y^2 = 169$ is a director circle having equation $x^2 + y^2 = 338$.

SECTION – III

Linked Comprehension Type

*This section contains 2 paragraphs M_{58-60} and M_{61-63}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choice (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

M_{58-60} : Paragraph for question Nos. 58 to 60

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

58. The ratio of the areas of the triangles PQS and PQR is

(A) $1 : \sqrt{2}$ (B) $1 : 2$
(C) $1 : 4$ (D) $1 : 8$

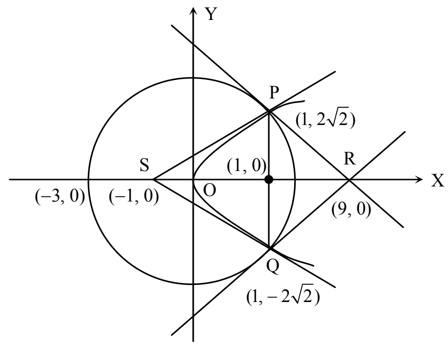
Sol. **(C)**

Coordinates of P and Q are $\left(1, +2\sqrt{2}\right)$ and $\left(1, -2\sqrt{2}\right)$.

Area of $\Delta PQR = \frac{1}{2}\cdot 4\sqrt{2}\cdot 8 = 16\sqrt{2}$

Area of $\Delta PQS = \frac{1}{2}\cdot 4\sqrt{2}\cdot 2 = 4\sqrt{2}$

Ratio of area of triangle PQS and PQR is 1 : 4.



59. The radius of the circumcircle of the triangle PRS is

(A) 5 (B) $3\sqrt{3}$

(C) $3\sqrt{2}$ (D) $2\sqrt{3}$

Sol. **(B)**

Equation of circumcircle of ΔPRS is $(x + 1)(x - 9) + y^2 + \lambda y = 0$

It will pass through $\left(1, 2\sqrt{2}\right)$, then $-16 + 8 + \lambda.\, 2\sqrt{2} = 0$

$\lambda = \frac{8}{2\sqrt{2}} = 2\sqrt{2}$

Equation of circumcircle is $x^2 + y^2 - 8x + 2\sqrt{2}y - 9 = 0$.

Hence its radius is $3\sqrt{3}$.

Alternative

Let $\angle PSR = \theta$

$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow PR = 6\sqrt{2} = 2R\cdot\sin\theta \Rightarrow R = 3\sqrt{3}$.

60. The radius of the incircle of the triangle PQR is

(A) 4 (B) 3

(C) 8/3 (D) 2

Sol. **(D)**

Radius of incircle is $r = \frac{\Delta}{s}$

as $\Delta = 16\sqrt{2}$

$s = \frac{6\sqrt{2} + 6\sqrt{2} + 4\sqrt{2}}{2} = 8\sqrt{2}$

$r = \frac{16\sqrt{2}}{8\sqrt{2}} = 2$.

M_{61-63} : Paragraph for Question Nos. 61 to 63

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is (2r − 1). Let

$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$

61. The sum $V_1 + V_2 + \ldots + V_n$ is

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

Sol. **(B)**

$V_r = \frac{r}{2}[2r + (r-1)(2r-1)] = \frac{1}{2}(2r^3 - r^2 + r)$

$\sum V_r = \frac{1}{12}n(n+1)(3n^2 + n + 2).$

62. T_r is always

(A) an odd number
(B) an even number
(C) a prime number
(D) a composite number

Sol. **(D)**

$V_{r+1} - V_r = (r+1)^3 - r^3 - \frac{1}{2}[(r+1)^2 - r^2] + \frac{1}{2}(1)$

$= 3r^2 + 2r + 1$

$T_r = 3r^2 + 2r - 1 = (r+1)(3r-1)$

which is a composite number.

63. Which one of the following is a correct statement?

(A) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5
(B) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6
(C) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11
(D) $Q_1 = Q_2 = Q_3 = \ldots$

Sol. **(B)**

$T_r = 3r^2 + 2r - 1$

$T_{r+1} = 3(r+1)^2 + 2(r+1) - 1$

$Q_r = T_{r+1} - T_r = 3[2r + 1] + 2[1]$

$Q_r = 6r + 5$

$Q_{r+1} = 6(r+1) + 5$

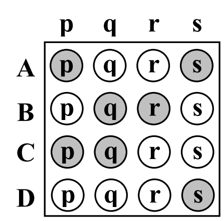
Common difference $= Q_{r+1} - Q_r = 6$.

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows:



64. Consider the following linear equations

$ax + by + cz = 0$
$bx + cy + az = 0$
$cx + ay + bz = 0$

Match the conditions / expressions in Column I with statements in Column II and indicate your answers by darkening the appropriate bubbles in 4 × 4 matrix given in the ORS.

Column I	Column II
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at a single point.
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

Sol. **A – r** **B – q** **C – p** **D – s**

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

(A). If $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow \Delta = 0$ and $a = b = c \neq 0$
$\Rightarrow$ the equations represent identical planes.

(B). $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
$\Rightarrow \Delta = 0 \Rightarrow$ the equations have infinitely many solutions.
$ax + by = (a + b)z$
$bx + cy = (b + c)z$
$\Rightarrow (b^2 - ac)y = (b^2 - ac)z \Rightarrow y = z$
$\Rightarrow ax + by + cy = 0 \Rightarrow ax = ay \Rightarrow x = y = z.$

(C). $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
$\Rightarrow \Delta \neq 0 \Rightarrow$ the equation represent planes meeting at only one point.

(D). $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca \Rightarrow a = b = c = 0$
$\Rightarrow$ the equation represent whole of the three dimensional space.

65. Match the integrals in Column I with the values in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) $\int_{-1}^{1} \frac{dx}{1+x^2}$	(p) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	(q) $2\log\left(\frac{2}{3}\right)$
(C) $\int_{2}^{3} \frac{dx}{1-x^2}$	(r) $\frac{\pi}{3}$
(D) $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	(s) $\frac{\pi}{2}$

Sol. **A – s** **B – s** **C – p** **D – r**

(A). $$\int_{-1}^{1} \frac{dx}{1+x^2} = \frac{\pi}{2}$$

(B). $$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2}$$

(C). $$\int_{2}^{3} \frac{dx}{1-x^2} = \frac{1}{2}\ln\frac{2}{3}$$

(D). $$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{3}$$

66. In the following [x] denotes the greatest integer less than or equal to x.
Match the functions in Column I with the properties Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	Column II
(A) $x\,\lvert x\rvert$	(p) continuous in $(-1, 1)$
(B) $\sqrt{\lvert x\rvert}$	(q) differentiable in $(-1, 1)$
(C) $x + [x]$	(r) strictly increasing in $(-1, 1)$
(D) $\lvert x-1\rvert + \lvert x+1\rvert$	(s) not differentiable at least at one point in $(-1, 1)$

Sol. **A – p, q, r** **B – p, s** **C – r, s** **D – p, q**

(A). $x\lvert x\rvert$ is continuous, differentiable and strictly increasing in $(-1, 1)$.

(B). $\sqrt{\lvert x\rvert}$ is continuous in $(-1, 1)$ and not differentiable at $x = 0$.

(C). $x + [x]$ is strictly increasing in $(-1, 1)$ and discontinuous at $x = 0 \Rightarrow$ not differentiable at $x = 0$.

(D). $\lvert x-1\rvert + \lvert x+1\rvert = 2$ in $(-1, 1)$
$\Rightarrow$ the function is continuous and differentiable in $(-1, 1)$.