FIITJEE Solutions to

IITJEE-2007

(Paper-II, Code-8)

Time: 3 hours ***M. Marks: 243***

Note: *(i)* The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.

(ii) **Section I** contains 9 multiple choice questions which have only one correct answer. Each question carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iii) **Section II** contains 4 questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason).
Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1
Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1
Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.
Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.
carries ***+3 marks*** each for correct answer and ***– 1 mark*** for each wrong answer.

(iv) **Section III** contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has only one correct answer and carries ***+4 marks*** for correct answer and ***– 1 mark*** for wrong answer.

(v) **Section IV** contains 3 questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column and each question carries ***+6 marks*** and marks will be awarded if all the four parts are correctly matched. No marks will be given for any wrong match in any question. There is no negative marking.

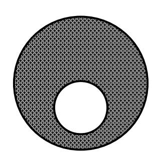
PART- I

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which ***ONLY ONE*** *is correct.*

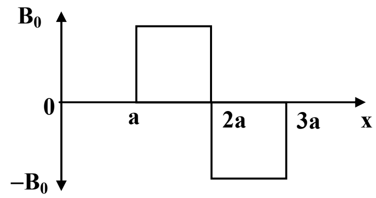
1. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in the figure. The electric field inside the emptied space is



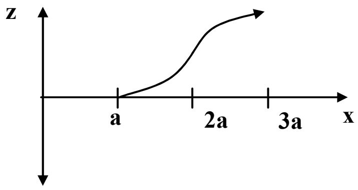
(A) zero everywhere (B) non-zero and uniform
(C) non-uniform (D) zero only at its center

Sol. **(B)**

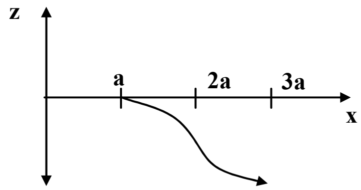
2. A magnetic field $\vec{B} = B_0\hat{j}$ exists in the region a < x < 2a and $\vec{B} = -B_0\hat{j}$, in the region 2a < x < 3a, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at x = a. The trajectory of the charge in this region can be like,



(A)



(B)



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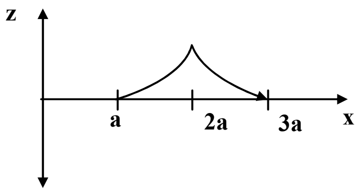
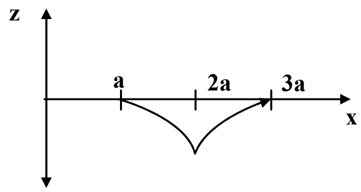


(D)

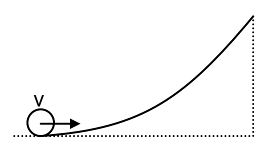


Sol. **(A)**

for a < x < 2a path will be concave upward

2a < x < 3a path will be concave downward

3. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is



(A) ring (B) solid sphere
(C) hollow sphere (D) disc

Sol. **(D)**

$$\frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$$

Hence $I_{cm} = \frac{1}{2}mR^2$

4. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

(A) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (B) $\lambda_0 = \frac{2h}{mc}$
(C) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0 = \lambda$

Sol. **(A)**

$$\lambda = \frac{h}{\sqrt{2m(eV)}} \Rightarrow eV = \frac{h^2}{2m\lambda^2}$$

$$\lambda_0 = \frac{hC}{eV}$$

5. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ±0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ±0.01 mm. Take g = 9.8 m/s^2 (exact). The Young's modulus obtained from the reading is
(A) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2
(B) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(C) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2
(D) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

Sol. **(B)**

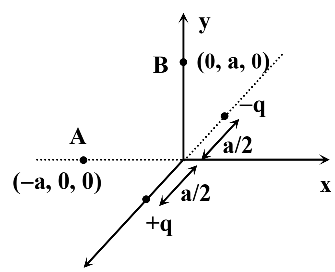
$$Y = \frac{4F\ell}{\pi d^2 \Delta\ell}$$

$$\frac{\Delta Y}{Y} = \frac{2\Delta D}{D} + \frac{\Delta(L)}{\Delta L} = 0.1125$$

$\Delta Y = 2 \times 10^{11} \times 0.1125$

6. Positive and negative point charges of equal magnitude are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, \frac{-a}{2}\right)$, respectively. The work done by the electric field when another positive point charge is moved from (–a, 0, 0) to (0, a, 0) is
(A) positive
(B) negative
(C) zero
(D) depends on the path connecting the initial and final positions

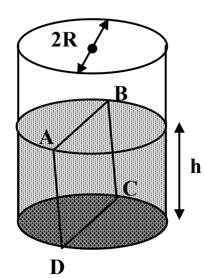
Sol. **(C)**



7. In the experiment to determine the speed of sound using a resonance column,
(A) prongs of the tuning fork are kept in a vertical plane
(B) prongs of the tuning fork are kept in a horizontal plane
(C) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
(D) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

Sol. **(A)**

8. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude

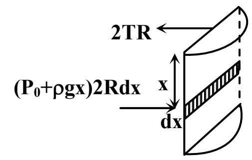


(A) $\left|2P_0Rh + \pi R^2\rho gh - 2RT\right|$
(B) $\left|2P_0Rh + R\rho gh^2 - 2RT\right|$
(C) $\left|P_0\pi R^2 + R\rho gh^2 - 2RT\right|$
(D) $\left|P_0\pi R^2 + R\rho gh^2 + 2RT\right|$

Sol. **(B)**

$$\int_0^h (P_0 + \rho g x)2R\,dx - 2RT = F$$

$2P_0Rh + R\rho gh^2 - 2RT = F$



9. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$, where A and k are constants. The angle between the force and the momentum is

(A) 0° (B) 30°
(C) 45° (D) 90°

Sol. **(D)**

$$\vec{F} = \frac{d\vec{P}}{dt} = AK\left[-\hat{i}\sin(kt) - \hat{j}\cos(kt)\right]$$

$$\vec{F}\cdot\vec{P} = 0$$

SECTION – II

Assertion - Reason Type

This section contains 4 questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

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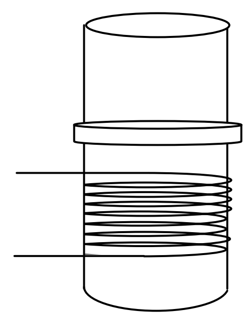
10. **STATEMENT-1**

A vertical iron rod has a coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.



Because

STATEMENT-2

In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction.

(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(A)**

11. **STATEMENT-1**

If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant.

because

STATEMENT-2

The linear momentum of an isolated system remains constant.

(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.

(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(D)**

12. **STATEMENT-1**
The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

because

STATEMENT-2

The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.
(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

13. **STATEMENT-1**
A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.
because

STATEMENT-2
For every action there is an equal and opposite reaction.
(A) Statement -1 is True, Statement-2 is True; Statement -2 **is** a correct explanation for Statement-1.
(B) Statement -1 is True, Statement-2 is True; Statement -2 **is NOT** a correct explanation for Statement-1.
(C) Statement -1 is True, Statement-2 is False.
(D) Statement -1 is False, Statement-2 is True.

Sol. **(B)**

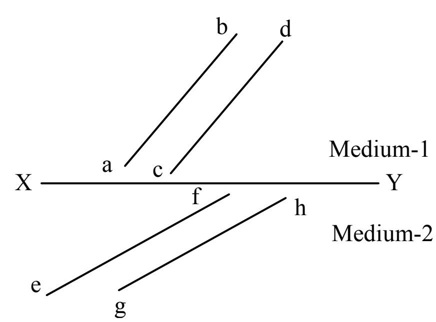
SECTION – III

Linked Comprehension Type

*This section contains 2 paragraphs $P_{14\text{-}16}$ and $P_{17\text{-}19}$. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

P_{14-16} : Paragraph for Question Nos. 14 to 16

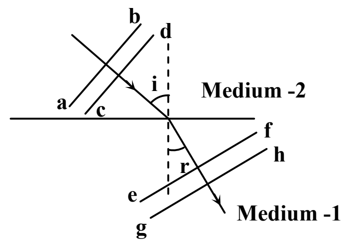
The figure shows a surface XY separating two transparent media, medium – 1 and medium -2. The lines ab and cd represent wavefronts of a light wave travelling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



14. Light travels as a
(A) parallel beam in each medium
(B) convergent beam in each medium

(C) divergent beam in each medium
(D) divergent beam in one medium and convergent beam in the other medium.

Sol. **(A)**



15. The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$
(A) ϕ_c cannot be equal to ϕ_d
(B) ϕ_d can be equal to ϕ_e
(C) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
(D) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

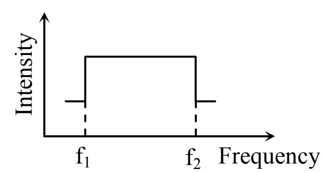
Sol. **(C)**

16. Speed of the light is
(A) the same in medium-1 and medium-2
(B) larger in medium-1 than in medium-2
(C) larger in medium-2 than in medium-1
(D) different at b and d

Sol. **(B)**

P_{17-19} : Paragraph for Question Nos. 17 to 19

Two trains A and B are moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.

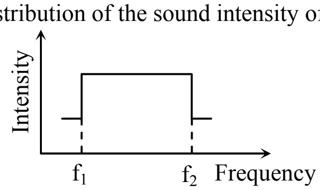


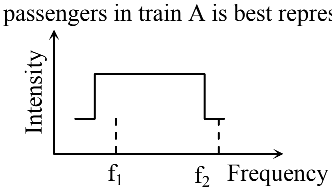
Assume that the sound of the whistle is composed of components varying in frequency from f_1 = 800 Hz to f_2 = 1120 Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.

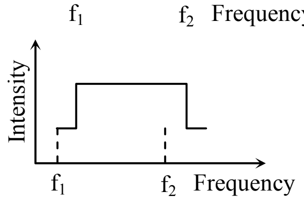
17. The speed of sound of the whistle is
(A) 340 m/s for passengers in A and 310 m/s for passengers in B
(B) 360 m/s for passengers in A and 310 m/s for passengers in B
(C) 310 m/s for passengers in A and 360 m/s for passengers in B
(D) 340 m/s for passengers in both the trains

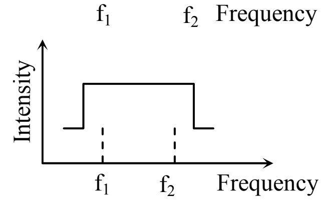
Sol. **(B)**
Speed of sound is frame dependent.

18. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by

(A)


(B)


(C)


(D)


Sol. **(A)**

19. The spread of frequency as observed by the passengers in train B is

(A) 310 Hz (B) 330 Hz
(C) 350 Hz (D) 290 Hz

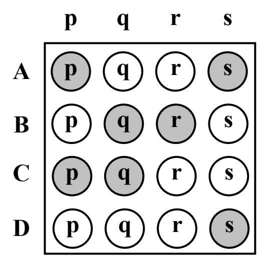
Sol. **(A)**

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



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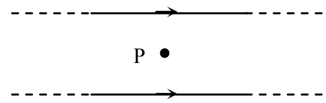
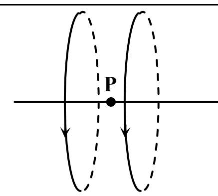
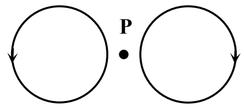
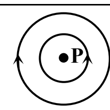
20. **Column I** describe some situations in which a small object moves. **Column II** describes some characteristics of these motions. Match the situation in **Column I** with the characteristics in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) The object moves on the x-axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where, M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

Sol. A→ (p), B → (q) & (r), C → (p), D → (r) & (q)

21. Two wires each carrying a steady current I are shown in four configurations in **Column I**. Some of the resulting effects are described in **Column II**. Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II

(A) Point P is situated midway between the wires.		(p) The magnetic fields (B) at P due to the currents in the wires are in the same direction.
(B) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.		(q) The magnetic fields (B) at P due to the currents in the wires are in opposite directions.
(C) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.		(r) There is no magnetic field at P.
(D) Point P is situated at the common center of the wires.		(s) The wires repel each other.

Sol. A→ (q) & (r), B → (p), C → (q) & (r), D → (q)

22. **Column I** gives some devices and **Column II** gives some process on which the functioning of these devices depend. Match the devices in **Column I** with the processes in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) Bimetallic strip	(p) Radiation from a hot body
(B) Steam engine	(q) Energy conversion
(C) Incandescent lamp	(r) Melting
(D) Electric fuse	(s) Thermal expansion of solids

Sol. A→ (s), B → (q), C → (p), D → (r)

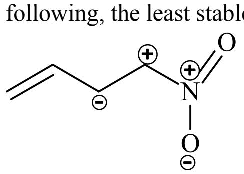
PART- II (CHEMISTRY)

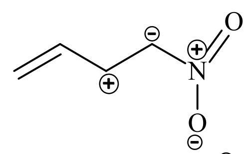
SECTION – I

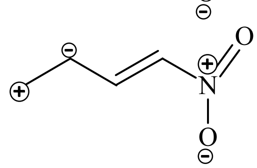
Straight Objective Type

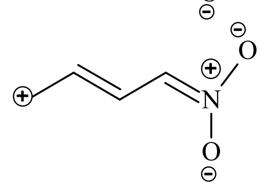
This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), (C) and (D), out of which ***ONLY ONE*** *is correct.*

23. Among the following, the least stable resonance structure is

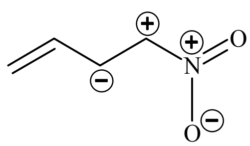
(A) 

(B) 

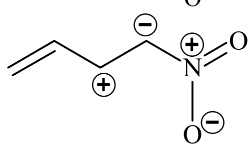
(C) 

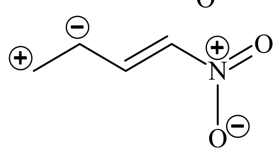
(D) 

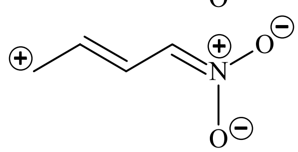
Sol. **(A)**

(A) 

Same charges are present at nearest position (Less stable)

(B) 

(C) 

(D) 

Hence (A) is correct

24. For the process $H_2O(l)$ (1 bar, 373 K) $\rightarrow$ $H_2O(g)$ (1 bar, 373 K), the correct set of thermodynamic parameters is

(A) $\Delta G = 0, \Delta S = +ve$ (B) $\Delta G = 0, \Delta S = -ve$

(C) $\Delta G = +ve, \Delta S = 0$ (D) $\Delta G = -ve, \Delta S = +ve$

Sol. **(A)**

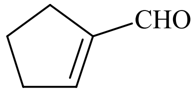
$$\underset{(1\text{ bar}, 373\text{ K})}{H_2O(\ell)} \rightleftharpoons \underset{(1\text{ bar}, 373\text{K})}{H_2O(g)}$$

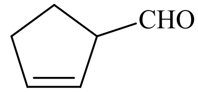
At 100°C $H_2O(\ell)$ has equilibrium with $H_2O(g)$ therefore $\Delta G = 0$.

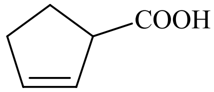
Because liquid molecules are converting into gases molecules therefore $\Delta S = +ve$

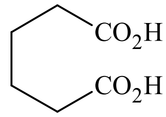
Hence (A) is correct.

25. Cyclohexene on ozonolysis followed by reaction with zinc dust and water gives compound E. Compound E on further treatment with aqueous KOH yields compound F. Compound F is

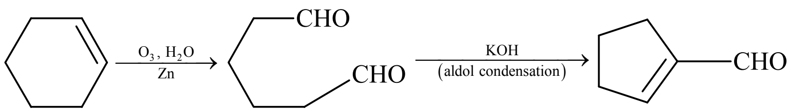
(A) 

(B) 

(C) 

(D) 

Sol. **(A)**



(E) (F)

Hence (A) is correct.

26. Consider a reaction aG + bH $\rightarrow$ Products. When concentration of both the reactants G and H is doubled, the rate increases by eight times. However, when concentration of G is doubled keeping the concentration of H fixed, the rate is doubled. The overall order of the reaction is

(A) 0
(B) 1
(C) 2
(D) 3

Sol. **(D)**

aG + bH $\longrightarrow$ Product

rate $\propto [G]^a [H]^b$

a = 1, b = 2

Hence (D) is correct

27. Among the following metal carbonyls, the C – O bond order is lowest in

(A) $[Mn(CO)_6]^+$
(B) $[Fe(CO)_5]$
(C) $[Cr(CO)_6]$
(D) $[V(CO)_6]^-$

Sol. **(B)**

(A) $Mn^+ = 3d^5 4s^1$ in presence of CO effective configuration = $3d^6 4s^0$.
Three lone pair for back bonding with vacant orbital of C in CO.

(B) $Fe^0 = 3d^6 4s^2$ in presence of CO effective configuration = $3d^8$ four lone pair for back bonding with CO.

(C) $Cr^0 = 3d^5 4s^1$
Effective configuration = $3d^6$.
Three lone pair for back bonding with CO.

(D) $V^- = 3d^4 4s^2$ effective configuration = $3d^6$ three lone pair for back bonding with CO.
Maximum back bonding is present in $Fe(CO)_5$ there for CO bond order is lowest here.

28. A positron is emitted from ${}^{23}Na_{11}$. The ratio of the atomic mass and atomic number of the resulting nuclide is

(A) 22/10
(B) 22/11
(C) 23/10
(D) 23/12

Sol. **(C)**

On position emission from nucleus, proton converts into neutron therefore atomic number decreases by one but atomic mass remains constant.

$$\frac{\text{Atomic mass}}{\text{atomic number}} = \frac{23}{10}$$

Hence (C) is correct.

29. Consider a titration of potassium dichromate solution with acidified Mohr's salt solution using diphenylamine as indicator. The number of moles of Mohr's salt required per mole of dichromate is

(A) 3
(B) 4
(C) 5
(D) 6

Sol. **(D)**

$Cr_2O_7^{2-} + Fe^{2+} \longrightarrow Fe^{3+} + Cr^{3+}$

n factor of $Cr_2O_7^{2-}$ = 6

n factor of Fe^{2+} = 1

So to reduce one mole of dichromate 6 moles of Fe^{2+} are required.

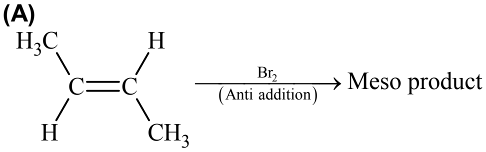
Hence (D) is correct.

30. The number of stereoisomers obtained by bromination of *trans*-2-butene is

(A) 1 (B) 2

(C) 3 (D) 4

Sol. **(A)**



Hence (A) is correct.

31. A solution of metal ion when treated with KI gives a red precipitate which dissolves in excess KI to give a colourless solution. Moreover, the solution of metal ion on treatment with a solution of cobalt(II) thiocyanate gives rise to a deep blue crystalline precipitate. The metal ion is

(A) Pb^{2+} (B) Hg^{2+}

(C) Cu^{2+} (D) Co^{2+}

Sol. **(B)**

$Hg^{2+} + KI \longrightarrow HgI_2 \text{ (Red ppt.)}$

$HgI_2 + \underset{(\text{excess})}{KI} \longrightarrow \underset{\text{Soluble}}{K_2HgI_4}$

$HgI^{2+} + Co(SCN)_2 \longrightarrow \underset{\text{blue crystalline precipiates}}{Hg(SCN)_2}$

SECTION – II

Assertion – Reason Type

This section 4 questions numbered 32 to 35. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

32. STATEMENT-1: Molecules that are not superimposable on their mirror images are chiral

because

STATEMENT-2: All chiral molecules have chiral centres.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol. **(C)**

33. STATEMENT-1: Alkali metals dissolves in liquid ammonia to give blue solution

because

STATEMENT-2: Alkali metals in liquid ammonia give solvated species of the type $[M(NH_3)_n]^+$ (M = alkali metals).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Sol. **(B)**

Blue colour is due to solvated electrons.

34. STATEMENT-1: Band gap in germanium is small.

because

STATEMENT-2: The energy spread of each germanium atomic energy level is infinitesimally small.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol. (C)

35. STATEMENT-1: Glucose gives a reddish-brown precipitate with Fehling's solution.
because
STATEMENT-2: Reaction of glucose with Fehling's solution gives CuO and gluconic acid.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol. (C)

$$C_6H_{12}O_6 + \text{Fehling solution} \longrightarrow (C_6H_{11}O_7)^- + \underset{\text{(Red ppt.)}}{Cu_2O\downarrow}$$

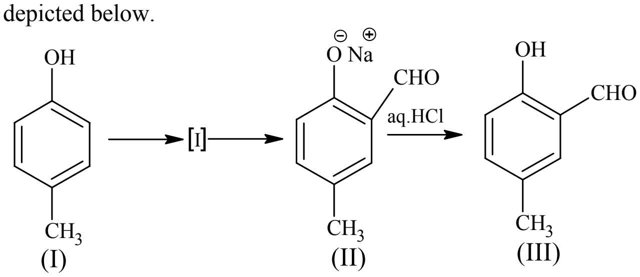
SECTION – III

Linked Comprehension Type

This section contains 2 paragraphs C_{36-38} and C_{39-41}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

C_{36-38} : Paragraph for question Nos 36 to 38

Reimer-Tiemann reaction introduces an aldehyde group, on to the aromatic ring of phenol, *ortho* to the hydroxyl group. This reaction involves electrophilic aromatic substitution. This is a general method for the synthesis of substituted salicyladehydes as depicted below.



36. Which one of the following reagents is used in the above reaction?
(A) aq. $NaOH + CH_3Cl$
(B) aq. $NaOH + CH_2Cl_2$
(C) aq. $NaOH + CHCl_3$
(D) aq. $NaOH + CCl_4$

Sol. (C)

$$\overset{\ominus}{OH}_{aq} + CHCl_3 \rightleftharpoons \overset{\ominus}{C}Cl_3 + H_2O$$

$$C\overset{\ominus}{Cl_3} \longrightarrow \overset{\ominus}{Cl} + :CCl_2$$

dichlorocarbene intermediate

37. The electrophile in this reaction is
(A) : CHCl
(B) $^{+}CHCl_2$
(C) $:CCl_2$
(D) $.CCl_3$

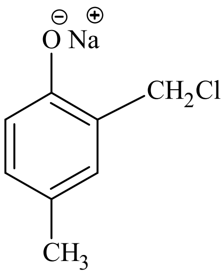
Sol. (C)

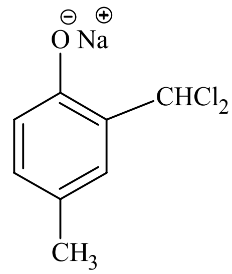
$$\overset{\ominus}{OH}_{aq} + CHCl_3 \rightleftharpoons \overset{\ominus}{C}Cl_3 + H_2O$$

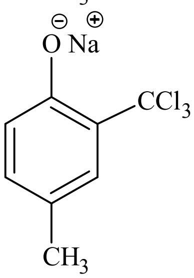
$$C\overset{\ominus}{Cl_3} \longrightarrow \overset{\ominus}{Cl} + :CCl_2$$

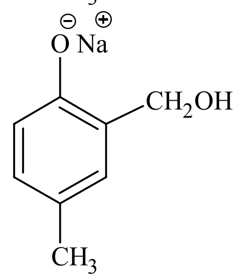
dichlorocarbene intermediate

38. The structure of the intermediate I is

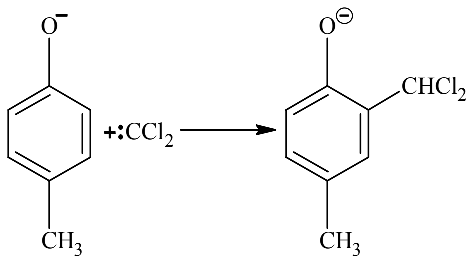
(A)


(B)


(C)


(D)


Sol. (B)



$C_{39\text{-}41}$: Paragraph for question Nos 39 to 41

Redox reactions play a pivotal role in chemistry and biology. The values of standard redox potential (E^o) of two half-cell reactions decide which way the reaction is expected to proceed. A simple example is a Daniel cell in which zinc goes into solution and copper gets deposited. Given below are a set of half-cell reactions (acidic medium) along with their E^o (V with respect to normal hydrogen electrode) values. Using this data obtain the correct explanations to Questions 39-41.

$I_2 + 2e^- \rightarrow 2I^-$ $E^o = 0.54$
$Cl_2 + 2e^- \rightarrow 2Cl^-$ $E^o = 1.36$
$Mn^{3+} + e^- \rightarrow Mn^{2+}$ $E^o = 1.50$
$Fe^{3+} + e^- \rightarrow Fe^{2+}$ $E^o = 0.77$
$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$ $E^o = 1.23$

39. Among the following, identify the correct statement.
(A) Chloride ion is oxidized by O_2
(B) Fe^{2+} is oxidized by iodine
(C) Iodide ion is oxidized by chlorine
(D) Mn^{2+} is oxidized by chlorine

Sol. (C)
Reduction potential of I_2 is less than Cl_2.

40. While Fe^{3+} is stable, Mn^{3+} is not stable in acid solution because
(A) O_2 oxidises Mn^{2+} to Mn^{3+}
(B) O_2 oxidises both Mn^{2+} and Fe^{2+} to Fe^{3+}
(C) Fe^{3+} oxidizes H_2O to O_2
(D) Mn^{3+} oxidises H_2O to O_2

Sol. (D)
Reaction of Mn^{3+} with H_2O is spontaneous.

41. Sodium fusion extract, obtained from aniline, on treatment with iron (II) sulphate and H_2SO_4 in presence of air gives a Prussian blue precipitate. The blue colour is due to the formation of
(A) $Fe_4[Fe(CN)_6]_3$
(B) $Fe_3[Fe(CN)_6]_2$
(C) $Fe_4[Fe(CN)_6]_2$
(D) $Fe_3[Fe(CN)_6]_3$

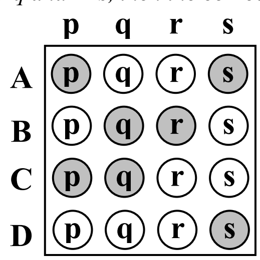
Sol. (A)

SECTION – IV

Matrix-Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (p, q, r, s) in Column-II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-q, B-r, C-p, C-q and D-s, then the correctly bubbled 4 × 4 matrix should be as follows:



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42. Match the reactions in Column I with nature of the reactions/type of the products in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) $O_2^- \rightarrow O_2 + O_2^{2-}$	(p) redox reaction
(B) $CrO_4^{2-} + H^+ \rightarrow$	(q) one of the products has trigonal planar structure
(C) $MnO_4^- + NO_2^- + H^+ \rightarrow$	(r) dimeric bridged tetrahedral metal ion
(D) $NO_3^- + H_2SO_4 + Fe^{2+} \rightarrow$	(s) disproportionation

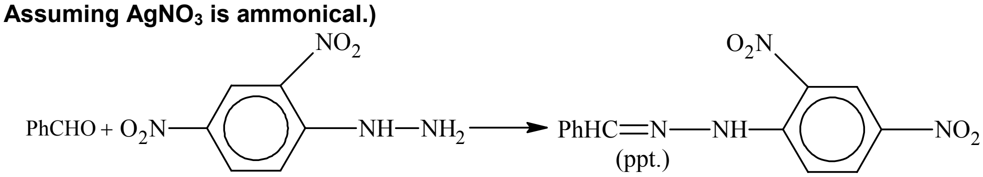
Sol. **A – p, s**
B –r
C – p, q
D – p

43. Match the compounds/ions in Column I with their properties/reactions in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

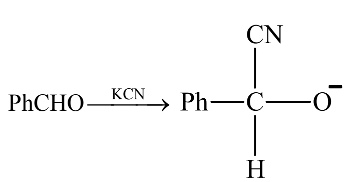
Column I	Column II
(A) C_6H_5CHO	(p) gives precipitate with 2, 4-dinitrophenylhydrazine
(B) $CH_3C \equiv CH$	(q) gives precipitate with $AgNO_3$
(C) CN^-	(r) is a nucleophile
(D) I^-	(s) is involved in cyanohydrin formation

Sol. **A – p, q, s**
B –q
C – q, r, s
D – q, r
(Note: Assuming $AgNO_3$ is ammonical.)

(A)

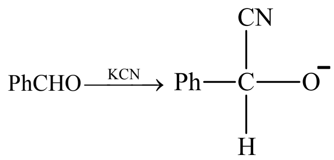


$$PhCHO + Ag_2O \xrightarrow{NH_3} PhCOO^- + \underset{\text{(white ppt.)}}{Ag\downarrow}$$



(B) $$CH_3C \equiv CH \xrightarrow{\text{ammonical } AgNO_3} \underset{\text{(White ppt.)}}{CH_3 - C \equiv C^-Ag^+ \downarrow}$$

(C)



$AgNO_3 + CN^- \longrightarrow AgCN \downarrow$

(D) $AgNO_3 + I^- \longrightarrow AgI \downarrow$

44. Match the crystal system/unit cells mentioned in Column I with their characteristic features mentioned in Column II. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) Simple cubic and face-centred cubic	(p) have these cell parameters $a = b = c$ and $\alpha = \beta = \gamma$
(B) cubic and rhombohedral	(q) are two crystal systems
(C) cubic and tetragonal	(r) have only two crystallography angles of 90^o
(D) hexagonal and monoclinic	(s) belong to same crystal system

Sol. **A – p, s**
B –p, q
C – q
D – q, r

Crystals class	Axial distances	Angles
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90°$
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90°$
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90°$
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90°$ $\gamma = 120°$
Trigonal and rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90°$
Monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90°$ $\gamma \neq 90°$
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$

PART- III (MATHEMATICS)

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

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45. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct?

(A) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$
(B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$
(C) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=\vec{0}$
(D) $\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ are mutually perpendicular

Sol. **(B)**

Since $\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a}+\vec{b}+\vec{c}=0$,

$\vec{a}, \vec{b}, \vec{c}$ represent an equilateral triangle.

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq 0$.

46. Let $f(x)=\dfrac{x}{(1+x^n)^{1/n}}$ for $n \ge 2$ and $g(x) = \underbrace{(fofo\ldots of)}_{f\text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2}g(x)dx$ equals

(A) $\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+K$
(B) $\dfrac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+K$
(C) $\dfrac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+K$
(D) $\dfrac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+K$

Sol. **(A)**

Here $ff(x)=\dfrac{f(x)}{[1+f(x)^n]^{1/n}}=\dfrac{x}{(1+2x^n)^{1/x}}$

$fff(x)=\dfrac{x}{(1+3x^n)^{1/n}}$

$\Rightarrow g(x) = \underbrace{(fofo\ldots of)}_{n\text{ terms}}(x) = \dfrac{x}{(1+nx^n)^{1/n}}$

Hence $I=\int x^{n-2}g(x)dx=\int\dfrac{x^{n-1}dx}{(1+nx^n)^{1/n}}$

$=\dfrac{1}{n^2}\int\dfrac{n^2x^{n-1}dx}{(1+nx^n)^{1/n}}=\dfrac{1}{n^2}\int\dfrac{\frac{d}{dx}(1+nx^n)}{(1+nx^n)^{1/n}}dx$

$\therefore\ I=\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+k$.

47. $\dfrac{d^2x}{dy^2}$ equals

(A) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$
(B) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$
(C) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$
(D) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

Sol. **(D)**

Since, $\frac{dx}{dy} = \frac{1}{dy/dx} = \left(\frac{dy}{dx}\right)^{-1}$

$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1} \frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}\left(\frac{dx}{dy}\right) = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$.

*48. The letters of the word **COCHIN** are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word **COCHIN** is

(A) 360 (B) 192
(C) 96 (D) 48

Sol. **(C)**
COCHIN
The second place can be filled in $^4C_1$ ways and the remaining four alphabets can be arranged in 4! ways in four different places. The next 97th word will be COCHIN.
Hence, there are 96 words before COCHIN.

*49. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on

(A) a line not passing through the origin (B) $|z| = \sqrt{2}$
(C) the x-axis (D) the y-axis

Sol. **(D)**
Let z = cosθ + sinθ, so that

$\frac{z}{1-z^2} = \frac{\cos\theta + \sin\theta}{1-(\cos 2\theta + i\sin 2\theta)}$

$= \frac{\cos\theta + i\sin\theta}{2\sin^2\theta - 2i\sin\theta\cos\theta} = \frac{\cos\theta + i\sin\theta}{-2i\sin\theta(\cos\theta + i\sin\theta)}$

$= \frac{i}{2\sin\theta}$

Hence $\frac{z}{1-z^2}$ lies on the imaginary axis i.e., x = 0.

Alternative

Let $E = \frac{z}{1-z^2} = \frac{z}{z\bar{z} - z^2} = \frac{1}{\bar{z} - z}$

which is imaginary.

*50. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

(A) 3 (B) 2
(C) 3/2 (D) 1

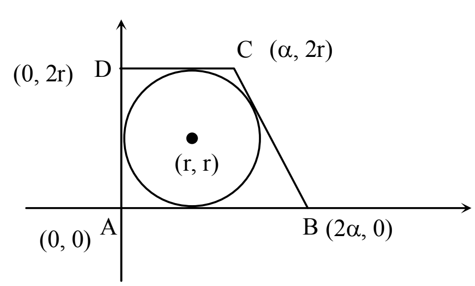
Sol. **(B)**

$18 = \frac{1}{2}(3\alpha)(2r) \Rightarrow \alpha r = 6$

Line $y = -\frac{2r}{\alpha}(x - 2\alpha)$ is tangent to $(x - r)^2 + (y - r)^2 = r^2$

$2\alpha = 3r$ and $\alpha r = 6$

r = 2.



Alternate

$\frac{1}{2}(x+2x)\times 2r = 18$

$xr = 6$ …(1)

$\tan\theta = \frac{x-r}{r}$ $\tan(90-\theta) = \frac{2x-r}{r}$

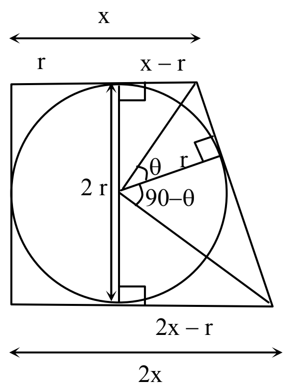
$\frac{x-r}{r} = \frac{r}{2x-r}$

$x(2x-3r) = 0$

$x = \frac{3r}{2}$ …(2)

From (1) and (2)

r = 2.



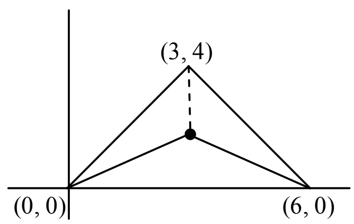
*51. Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$

(C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

Sol. **(C)**

Since, Δ is isosceles, hence centroid is the desired point.



52. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at (0, 1)
(B) variable radii and a fixed centre at (0, –1)
(C) fixed radius 1 and variable centres along the x-axis
(D) fixed radius 1 and variable centres along the y-axis

Sol. **(C)**

$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

$\Rightarrow \int \frac{y}{\sqrt{1-y^2}} dy = \int dx$

$\Rightarrow -\sqrt{1-y^2} = x + c$

$\Rightarrow (x+c)^2 + y^2 = 1$

centre (–c, 0); radius $\sqrt{c^2 - c^2 + 1} = 1$.

53. Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with P(G) > 0 and $P(E\cap F\cap G) = 0$. Then $P(E^C\cap F^C \mid G)$ equals

(A) $P(E^C) + P(F^C)$ (B) $P(E^C) - P(F^C)$

(C) $P(E^C) - P(F)$ (D) $P(E) - P(F^C)$

Sol. **(C)**

$P\left(\frac{E^c \cap F^c}{G}\right) = \frac{P(E^c \cap F^c \cap G)}{P(G)} = \frac{P(G) - P(E\cap G) - P(G\cap F)}{P(G)}$

$= \dfrac{P(G)(1 - P(E) - P(F))}{P(G)}$ $\quad [\because P(G) \neq 0]$

$= 1 - P(E) - P(F)$

$= P(E^c) - P(F)$.

SECTION –II

Assertion – Reason Type

*This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT – 1 (Assertion) and STATEMENT -2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.*

54. Let $f(x) = 2 + \cos x$ for all real x.

STATEMENT -1 : For each real t, there exists a point c in $[t, t + \pi]$ such that $f'(c) = 0$.

because

STATEMENT -2 : $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(B)**
$f(x) = 2 + \cos x \ \forall \ x \in R$
Statement : 1
There exists a point $c \in [t, t + \pi]$ where $f'(c) = 0$
Hence, statement 1 is true.
Statement 2:
$f(t) = f(t + 2\pi)$ is true.
But statement 2 is not a correct explanation for statement 1.

55. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

STATEMENT -1 : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$

because

STATEMENT -2 : The vectors $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of the given planes.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(D)**
$3x - 6y - 2z = 15$
$2x + y - 2z = 5$
for $z = 0$, we get $x = 3$, $y = -1$
Direction vectors of plane are
$< 3 \ \ -6 \ \ -2 >$ and $<2 \ \ 1 \ \ -2>$
then the dr's of line of intersection of planes is $<14 \ \ 2 \ \ 15>$

$$\frac{x-3}{14} = \frac{y+1}{2} = \frac{z-0}{15} = \lambda$$

$\Rightarrow x = 14\lambda + 3 \quad y = 2\lambda - 1 \quad z = 15\lambda$
Hence, statement 1 is false.
But statement 2 is true.

*56. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

STATEMENT -1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

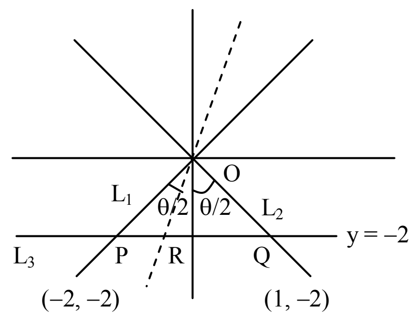
because

STATEMENT -2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.
(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(C)**

In ΔOPQ

Clearly $\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$



*57. STATEMENT -1 : The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line x = 1.

because

STATEMENT -2 : A parabola is symmetric about its axis.
(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1
(B) Statement -1 is True, Statement -2 is true; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement -1 is True, Statement -2 is False
(D) Statement -1 is False, Statement -2 is True

Sol. **(A)**

$y = -\frac{x^2}{2} + x + 1$

$\Rightarrow y - \frac{3}{2} = -\frac{1}{2}(x-1)^2$

$\Rightarrow$ it is symmetric about x = 1.

SECTION – III

Linked Comprehension Type

*This section contains 2 paragraphs M_{58-60} and M_{61-63}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choice (A), (B), (C) and (D), out of which **ONLY ONE** is correct.*

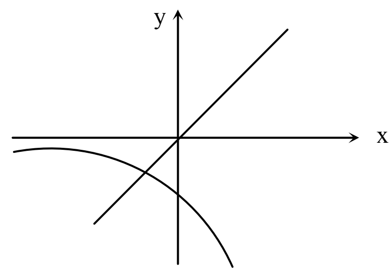
M_{58-60} : Paragraph for question Nos. 58 to 60

If a continuous f defined on the real line R, assumes positive and negative values in R then the equation f(x) = 0 has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative then the equation f(x) = 0 has a root in R.
Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

58. The line y = x meets $y = ke^x$ for $k \le 0$ at

(A) no point (B) one point
(C) two points (D) more than two points

Sol. **(B)**
Line y = x intersect the curve $y = ke^x$ at exactly one point when $k \le 0$.



59. The positive value of k for which $ke^x - x = 0$ has only one root is
(A) 1/e (B) 1
(C) e (D) $\log_e 2$

Sol. **(A)**
Let $f(x) = ke^x - x$
$f'(x) = ke^x - 1 = 0 \Rightarrow x = -\ln k$
$f''(x) = ke^x$
$f''(x)|_{x=-\ln k} = 1 > 0$
Hence $f(-\ln k) = 1 + \ln k$
For one root of given equation
$1 + \ln k = 0$
hence $k = \frac{1}{e}$.

60. For k > 0, the set of all values of k for which $k e^x - x = 0$ has two distinct roots is
(A) $\left(0, \frac{1}{e}\right)$ (B) $\left(\frac{1}{e}, 1\right)$
(C) $\left(\frac{1}{e}, \infty\right)$ (D) (0, 1)

Sol. **(A)**
For two distinct roots $1 + \ln k < 0$ $(k > 0)$
$\ln k < -1$
$k < \frac{1}{e}$
hence $k \in \left(0, \frac{1}{e}\right)$.

M_{61-63} : Paragraph for Question Nos. 61 to 63

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \ge 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

*61. Which one of the following statements is correct?
(A) $G_1 > G_2 > G_3 > \ldots$ (B) $G_1 < G_2 < G_3 < \ldots$
(C) $G_1 = G_2 = G_3 = \ldots$ (D) $G_1 < G_3 < G_5 < \ldots$ and $G_2 > G_4 > G_6 > \ldots$

Sol. **(C)**
$A_1 = \frac{a+b}{2}; G_1 = \sqrt{ab}; H_1 = \frac{2ab}{a+b}$
$A_n = \frac{A_{n-1} + H_{n-1}}{2}, G_n = \sqrt{A_{n-1}H_{n-1}}, H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$

Clearly, $G_1 = G_2 = G_3 = \ldots = \sqrt{ab}$.

*62. Which of the following statements is correct?
(A) $A_1 > A_2 > A_3 > \ldots$
(B) $A_1 < A_2 < A_3 < \ldots$
(C) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$
(D) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

Sol. **(A)**
A_2 is A.M. of A_1 and H_1 and $A_1 > H_1 \Rightarrow A_1 > A_2 > H_1$
A_3 is A.M. of A_2 and H_2 and $A_2 > H_2 \Rightarrow A_2 > A_3 > H_2$
$\therefore A_1 > A_2 > A_3 > \ldots$

*63. Which of the following statements is correct?
(A) $H_1 > H_2 > H_3 > \ldots$
(B) $H_1 < H_2 < H_3 < \ldots$
(C) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 < \ldots$
(D) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

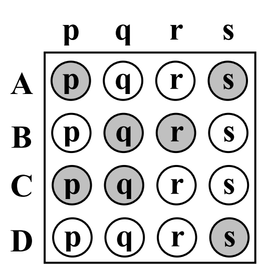
Sol. **(B)**
As above $A_1 > H_2 > H_1$, $A_2 > H_3 > H_2$
$\therefore H_1 < H_2 < H_3 < \ldots$

SECTION –IV

Matrix – Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A–p, A–s, B–q, B–r, C–p, C–q and D–s, then the correctly bubbled 4 × 4 matrix should be as follows:



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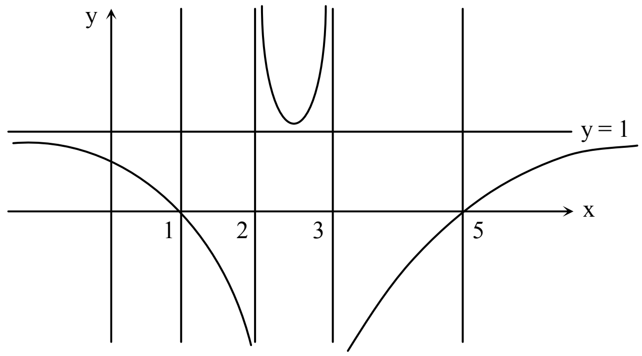
64. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$.
Match the conditions / expressions in **Column I** with statements in **Column II** and indicate your answers by darkening the appropriate bubbles in 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) If $-1 < x < 1$, then f(x) satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then f(x) satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then f(x) satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then f(x) satisfies	(s) $f(x) < 1$

Sol. **A → p, r, s ; B → q, s ; C → q, s ; D → p, r, s**

$f(x) = \dfrac{(x-1)(x-5)}{(x-2)(x-3)}$

The graph of f(x) is shown



(A) If $-1 < x < 1$
$\Rightarrow 0 < f(x) < 1$

(B) If $1 < x < 2 \Rightarrow f(x) < 0$

(C) If $3 < x < 5 \Rightarrow f(x) < 0$

(D) If $x > 5 \Rightarrow 0 < f(x) < 1$

*65. Let (x, y) be such that

$$\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}.$$

Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	Column II
(A) If a = 1 and b = 0, then (x, y)	(p) lies on the circle $x^2 + y^2 = 1$
(B) If a = 1 and b = 1, then (x, y)	(q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If a = 1 and b = 2, then (x, y)	(r) lies on $y = x$
(D) If a = 2 and b = 2, then (x, y)	(s) lies on $(4x^2 - 1)(y^2 - 1) = 0$

Sol. **A → p ; B → q ; C → p ; D → s**

(A) If a = 1, b = 0
then $\sin^{-1}x + \cos^{-1}y = 0$
$\Rightarrow \sin^{-1}x = -\cos^{-1}y$
$\Rightarrow x^2 + y^2 = 1.$

(B) If a = 1 and b = 1, then
$\sin^{-1}x + \cos^{-1}y + \cos^{-1}xy = \frac{\pi}{2}$
$\Rightarrow \cos^{-1}x - \cos^{-1}y = \cos^{-1}xy$
$\Rightarrow xy + \sqrt{1-x^2}\sqrt{1-y^2} = xy$ (taking sine on both the sides)

(C) If a = 1, b = 2
$\Rightarrow \sin^{-1}x + \cos^{-1}y + \cos^{-1}(2xy) = \frac{\pi}{2}$
$\Rightarrow \sin^{-1}x + \cos^{-1}y = \sin^{-1}(2xy)$
$\Rightarrow xy + \sqrt{1-x^2}\sqrt{1-y^2} = 2xy$
$\Rightarrow x^2 + y^2 = 1$ (on squaring).

(D) If a = 2 and b = 2 then
$\sin^{-1}(2x) + \cos^{-1}(y) + \cos^{-1}(2xy) = \frac{\pi}{2}$
$\Rightarrow 2xy + \sqrt{1-4x^2}\sqrt{1-y^2} = 2xy$
$\Rightarrow (4x^2 - 1)(y^2 - 1) = 0.$

*66. Match the statements in **Column I** with the properties **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the **ORS**.

Column I		**Column II**	
(A)	Two intersecting circles	(p)	have a common tangent
(B)	Two mutually external circles	(q)	have a common normal
(C)	two circles, one strictly inside the other	(r)	do not have a common tangent
(D)	two branches of a hyperbola	(s)	do not have a common normal

Sol. **A → p, q ; B → p, q ; C → q, r ; D → q, r**

(A) When two circles are intersecting they have a common normal and common tangent.
(B) Two mutually external circles have a common normal and common tangent.
(C) When one circle lies inside of other then, they have a common normal but no common tangent.
(D) Two branches of a hyperbola have a common normal but no common tangent.